MagnetoHydroDynamics (for stellarators)

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Great references
PART I: DESCRIBING A FUSION PLASMA
**Method I: Self-consistent particle pushing**

Natural idea: Move each particle according to \( F_p = m_p a_p \)

- **Difficulty 1:** There are MANY particles, \( N \sim 10^{20} - 10^{22} \) in magnetic fusion grade plasmas
- **Difficulty 2:** \( F_p \) depends on the position and velocity of all the other particles. \( F_p \) is expensive to compute e.g.: for electrostatic electric field force

\[
F_p = q_p \sum_{j=1}^{N} \frac{1}{4\pi \varepsilon_0} \frac{q_j}{|x_j - x_p|^2}
\]

- Problem still not tractable even with the most powerful computers when \( N \sim 10^{20} - 10^{22} \) and best algorithms
Even if computers could solve this problem, should we ask them to?
Debye shielding
Debye shielding

- Local charge imbalance shielded within a few $\lambda_D$
- $\lambda_D = \frac{e_0 T}{e^2 n}$ is called the Debye length
Method II: For weakly coupled plasmas, coarse-grain average in phase space

- Weakly coupled plasma: large \# of particles in any volume of size $\lambda_D^3$
- A large fraction of scientifically interesting plasmas are weakly coupled
- For weakly coupled plasmas, replace the discrete particles with smooth distribution function $f(x, v, t)$ defined so that

$$f(x, v, t)dx dv = \# \text{ of particles in } 6D \text{ phase-space volume } dx dv$$
Distribution function and Vlasov equation

- Macroscopic (fluid) quantities are velocity moments of $f$

  $$n(x, t) = \int \int \int f(x, v, t)dv$$  
  Density

  $$nV(x, t) = \int \int \int vf(x, v, t)dv$$  
  Mean flow

  $$P(x, t) = m \int \int \int (v - V)(v - V)f dv$$  
  Pressure tensor

- Conservation of $f$ along the phase-space trajectories of the particles determines the time evolution of $f$:

  $$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{dx}{dt} \cdot \nabla f + \frac{dv}{dt} \cdot \nabla vf = 0$$

  $$\frac{dx}{dt} = v \quad \frac{dv}{dt} = \frac{q}{m} (E + v \times B)$$

  $$\Rightarrow \quad \frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{q}{m} (E + v \times B) \cdot \nabla vf = 0$$

This is the Vlasov equation
THE BOLTZMANN EQUATION

- In fusion plasmas, we separate, leading to the Boltzmann equation:

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla \mathbf{v} f = \left( \frac{\partial f}{\partial t} \right)_c
\]

This equation to be combined with Maxwell’s equations:

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}
\]

- **Nonlinear, integro-differential, 6-dimensional PDE – Challenging**

- Describes phenomena on widely varying length \((10^{-5} - 10^3 \text{ m})\) and time \((10^{-12} - 10^2 \text{ s})\) scales

- Still not a piece of cake, and never solved as such for fusion plasmas
Simulation run on NERSC Edison supercomputer
Each simulation required 17000 processors and \( \sim 37 \) days (\( \sim 15 \)M CPU hours)
Work by N. Howard \textit{et al.} (MIT PSFC)
**Moment approach**

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{v} f = \left( \frac{\partial f}{\partial t} \right)_{c}
\]

- Taking the integrals \( \iiint d\mathbf{v} \), \( \iiint m \mathbf{v} d\mathbf{v} \) and \( \iiint m \mathbf{v}^2/2 d\mathbf{v} \) of this equation, we obtain the exact **fluid equations**:

  \[
  \frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{V}_s) = 0 \quad \text{Continuity}
  \]

  \[
  mn \left( \frac{\partial \mathbf{V}_s}{\partial t} + \mathbf{V}_s \cdot \nabla \mathbf{V}_s \right) = q_s n_s (\mathbf{E} + \mathbf{V}_s \times \mathbf{B}) - \nabla \cdot \mathbf{P}_s + \mathbf{R}_s \quad \text{Momentum}
  \]

  \[
  \frac{d}{dt} \left( \frac{3}{2} p_s \right) + \frac{5}{2} p_s \nabla \cdot \mathbf{V}_s + \pi_s : \nabla \mathbf{V}_s + \nabla \cdot \mathbf{q}_s = 0 \quad \text{(Energy)}
  \]

  with \( \mathbf{P}_s = p_s \mathbf{I} + \pi_s \).

- **Closure problem**: for each moment, we introduce a new unknown \( \Rightarrow \) End up with too many unknowns

- Need to make **approximations** to close the moment hierarchy
Kinetic models vs Fluid models

- For some fusion applications/plasma regimes (heating and current drive, transport), kinetic treatment cannot be avoided.
- Simplify and reduce dimensionality of the Vlasov equation with approximations:
  - Strong magnetization: Gyrokinetic equation
  - Small gyroradius compared to relevant length scales: Drift kinetic equation
  - Vanishing gyroradius: Kinetic MHD

- In contrast, fluid models are based on approximate expressions for higher order moments (off-diagonal entries in pressure tensor, heat flux) in terms of lower order quantities (density, velocity, diagonal entries in pressure tensor).

- We will now focus on the relevant regime and the approximations made to derive a widely used fluid model: the ideal MHD model.
PART II: THE IDEAL MHD MODEL
PHILOSOPHY OF IDEAL MHD

- The purpose of ideal MHD is to study the macroscopic behavior of the plasma.
- Use ideal MHD to design machines with desirable equilibrium magnetic configuration:
  - Reasonable coil/engineering requirements
  - Good transport properties
  - Avoids large scale instabilities
- Regime of interest:
  - Typical length scale: the minor radius of the device $a \sim 1m$
    - Wave number $k$ of waves and instabilities considered: $k \sim 1/a$
  - Typical velocities: Ion thermal velocity speed $v_T \sim 500km/s$
  - Typical time scale: $\tau_{MHD} \sim a/v_T \sim 2\mu s$
    - Frequency $\omega_{MHD}$ of associated waves/instabilities $\omega_{MHD} \sim 500kHz$
- Model cannot be used to study RF wave physics (heating, current drive), slow particle transport, and microinstabilities/microturbulence.
**Ideal MHD - Maxwell’s equations**

- $a \gg \lambda_D$, the distance over which charge separation can take place in a plasma
  - $\Rightarrow$ On the MHD length scale, the plasma is neutral: $n_i = n_e$

- $\omega_{\text{MHD}}/k \ll c$ and $v_{T_i} \ll v_{T_e} \ll c$ so we can neglect the displacement current in Maxwell’s equations:

\[
\begin{align*}
n_i &= n_e \quad (\Rightarrow \quad \nabla \cdot \mathbf{J} = 0) \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J}
\end{align*}
\]
Ideal MHD - Momentum Equation

- $a \gg \lambda_D$ and $a \gg r_{Le}$ (electron Larmor radius)
- $\omega_{MHD} \ll \omega_{pe}, \omega_{MHD} \ll \omega_{ce}$
- The ideal MHD model assumes that on the time and length scales of interest, the electrons have an infinitely fast response time to changes in the plasma
- Mathematically, this can be done by taking the limit $m_e \to 0$
- Adding the ion and electron momentum equation, we then get

$$\rho \frac{dV}{dt} - J \times B + \nabla p = -\nabla \cdot (\pi_i + \pi_e)$$

where $\rho = m_i n$ and $V$ is the ion fluid velocity
- If the condition $v_{Ti} \tau_{ii}/a \ll 1$ is satisfied in the plasma

$$\rho \frac{dV}{dt} = J \times B - \nabla p$$  (Ideal MHD momentum equation)
**Ideal MHD - Electrons**

- In the limit $m_e \rightarrow 0$, the electron momentum equation can be written as

\[
E + V \times B = \frac{1}{en} (J \times B - \nabla p_e - \nabla \cdot \pi_e + R_e)
\]

- This is called the generalized Ohm’s law

- Different MHD models (resitive MHD, Hall MHD) keep different terms in this equation

- If $r_{Li}/a \ll 1$, $\nu_{Ti} \tau_{ii}/a \ll 1$, and $(m_e/m_i)^{1/2}(r_{Li}/a)^2(a/\nu_{Ti} \tau_{ii}) \ll 1$, the momentum equation becomes the ideal Ohm’s law

\[
E + V \times B = 0
\]

- The ideal MHD plasma behaves like a perfectly conducting fluid
Energy Equation

- Define the total plasma pressure $p = p_i + p_e$

- Add electron and ion energy equations

- Under the conditions $r_{Li}/a \ll 1$ and $v_{Ti}\tau_{ii}/a \ll 1$, this simplifies as

$$\frac{d}{dt} \left( \frac{p}{\rho^{5/3}} \right) = 0$$

- Equation reminiscent of $pV^\gamma = Cst$: the ideal MHD plasma behaves like a monoatomic ideal gas undergoing a reversible adiabatic process
**Ideal MHD - Summary**

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0
\]

\[
\rho \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p
\]

\[
\frac{d}{dt} \left( \frac{p}{\rho^{5/3}} \right) = 0
\]

\[
\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
\]

\[
\nabla \times \mathbf{B} = \mu_0 \mathbf{J}
\]

\[
\nabla \cdot \mathbf{B} = 0
\]

Valid under the conditions

\[
\left( \frac{m_i}{m_e} \right)^{1/2} \left( \frac{v_i \tau_{ii}}{a} \right) \ll 1 \quad \frac{r_{Li}}{a} \ll 1 \quad \left( \frac{r_{Li}}{a} \right)^2 \left( \frac{m_e}{m_i} \right)^{1/2} \frac{a}{v_{Ti} \tau_{ii}} \ll 1
\]
VALIDITY OF THE IDEAL MHD MODEL (I)

- Are the conditions for the validity of ideal MHD
  
  \[
  \left( \frac{m_i}{m_e} \right)^{1/2} \left( \frac{\nu_{ii}}{a} \right) \ll 1 \quad \frac{r_{Li}}{a} \ll 1 \quad \left( \frac{r_{Li}}{a} \right)^2 \left( \frac{m_e}{m_i} \right)^{1/2} \frac{a}{\nu_{Ti} \tau_{ii}} \ll 1
  \]

  mutually compatible?

- Define \( x = (m_i/m_e)^{1/2}(\nu_{Ti} \tau_{ii}/a) \), \( y = r_{Li}/a \).

  \( x \ll 1 \) (High collisionality) \( y \ll 1 \) (Small ion Larmor radius)

  \( y^2/x \ll 1 \) (Small resistivity)

There exists a regime for which ideal MHD is justified (Figure from *Ideal MHD* by J.P. Freidberg, CUP, 2014)

Is that the regime of magnetic confinement fusion?
Validity of the Ideal MHD Model (II)

- Express three conditions in terms of usual physical parameters: $n, T, a$

- For tokamak-like pressures and $a = 1m$, we find:

  The regime of validity of ideal MHD does NOT coincide with the fusion plasma regime (Figure from *Ideal MHD* by J.P. Freidberg, CUP, 2014)

  The collisionality of fusion plasmas is too low for the ideal MHD model to be valid.

Is that a problem?
Validity of the ideal MHD model (III)

- It turns out that ideal MHD often does a very good job at predicting stability limits for macroscopic instabilities.
- This is not due to luck but to subtle physical reasons.
- One can show that collisionless kinetic models for macroscopic instabilities are more optimistic than ideal MHD.
- This is because ideal MHD is accurate for dynamics perpendicular to the fields lines.
- Designs based on ideal MHD calculations are conservative designs.
**FROZEN IN LAW**

- $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \mathbf{0}$: in the frame moving with the plasma, the electric field is zero

- The plasma behaves like a perfect conductor

- The magnetic field lines are "frozen" into the plasma motion

- $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B})$
Magnetic reconnection: a key phenomenon in astrophysical, space, and fusion plasmas

- Cannot happen according to ideal MHD
- Need to add additional terms in Ohm’s law to allow reconnection: resistivity, off-diagonal pressure tensor terms, electron inertia, ...

\[
\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{1}{ne} \nabla \cdot \mathbf{P}_e + \frac{m_e}{ne^2} \frac{\partial \mathbf{J}}{\partial t}
\]

\[
\Rightarrow \frac{\partial \mathbf{B}}{\partial t} \neq \nabla \times (\mathbf{V} \times \mathbf{B})
\]

- Associated instabilities take place on longer time scales than \( \tau_{MHD} \)
PART III: MHD EQUILIBRIUM
**EQUILIBRIUM STATE**

- By equilibrium, we mean steady-state: $\partial / \partial t = 0$
- Often, for simplicity and/or physical reasons, we focus on static equilibria: $\mathbf{V} = 0$

\[
\nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \mathbf{J} \times \mathbf{B} = \nabla p
\]

Note that the density profile does not appear

A key subtlety and source of difficulty: the radial profile of $p$ is given as input

- Implies the following intuitive form for force balance perpendicular to the magnetic field lines

\[
\nabla_\perp \left( p + \frac{B^2}{2\mu_0} \right) - \frac{B^2}{\mu_0} \kappa = 0 \quad \text{with} \quad \kappa = \mathbf{b} \cdot \nabla \mathbf{b}
\]

plasma and magnetic pressure  \hspace{1cm} field line tension
**TOROIDAL CONFINEMENT**

\[
(J \times B = \nabla p) \cdot B \Rightarrow B \cdot \nabla p = 0
\]

Magnetic field is tangent to surfaces of constant pressure

\[
(J \times B = \nabla p) \cdot J = 0 \Rightarrow J \cdot \nabla p = 0
\]

Current density is tangent to surfaces of constant pressure

*There is no nonvanishing continuous tangent vector field on an even-dimensional n-sphere*\(^1\)

Our confinement device MUST BE TOROIDAL

Virial theorem: For an MHD equilibrium to exist, the plasma must be held in force balance by externally supplied currents; it is not possible to create a configuration confined solely by the currents flowing within the plasma itself.
Why stellarators? Toroidal force balance

Toroidal field only

Solutions

Net outward “tire-tube” and $1/R$ force

Tokamak

Stellarator

Figure from T. Klinger et al., PPCF 59, 014018 (2017)
**ENERGY PRINCIPLE**

Ideal MHD equilibrium minimizes the quantity

\[ E \equiv \int_V \left( \frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) dV = E_{\text{magnetic}} + E_{\text{thermal}} \]

subject to the constraints of magnetic flux and entropy conservation

\[ \delta B = \nabla \times (\delta \xi \times B) \text{ (flux)}, \quad \delta p = -\delta \xi \cdot \nabla p - \gamma p \nabla \cdot \delta \xi \text{ (entropy)} \]

and boundary conditions

\[ B \cdot n \bigg|_{\partial V} = 0, \quad \delta \xi \cdot n \bigg|_{\partial V} = 0 \quad \text{fixed boundary calculation} \]

Indeed (DIY),

\[ \delta E = \int_V \delta \xi \cdot (\nabla p - J \times B) dV + \int_V \nabla \cdot \left( \frac{B \cdot \delta \xi B}{\mu_0} - \frac{\gamma}{\gamma - 1} p \delta \xi \right) dV \]
**FLUX SURFACES...OR NOT**

**Field line equations**

\[
\frac{d\psi_t}{d\phi} = \frac{B \cdot \nabla \psi}{B \cdot \nabla \phi} = -\frac{\partial \psi(\theta, \psi, \phi)}{\partial \theta} \\
\frac{d\psi_t}{d\phi} = \frac{B \cdot \nabla \phi}{B \cdot \nabla \psi} = \frac{\partial \psi(\theta, \psi, \phi)}{\partial \psi}
\]

Driven pendulum

Hamilton’s equations for simple pendulum

\[
\frac{dp_\theta}{dt} = -\frac{\partial H(\theta, p_\theta, t)}{\partial \theta} \\
\frac{d\theta}{dt} = \frac{\partial H(\theta, p_\theta, t)}{\partial p_\theta}
\]

**Stellarator**

Zero amplitude

Small amplitude

Large amplitude

Axisymmetric (tokamak)  Small nonaxisymmetry  Large nonaxisymmetry
CURRENT SHEETS – PART 1

Assume good flux surfaces, and write $B$ in straight field-line coordinates:

$$B = \nabla \psi_t \times \nabla \theta - \iota \nabla \psi_t \times \nabla \phi$$

$\iota = d\theta/d\phi$ is called the rotational transform

$\iota$ indicates how many poloidal turns a field line makes during each toroidal turn around the flux surface.

$\iota$ rational number $\Rightarrow$ closed field lines

Let $J = uB + J_\perp$, $\nabla \cdot J = 0$ $\Rightarrow$ $B \cdot \nabla u = -\nabla \cdot J_\perp$ \hspace{1cm} (1)

In straight field line coordinates, $\mathcal{J}B \cdot \nabla = i\partial_\theta + \partial_\phi$

Solve (1) by Fourier analysis:

$$(\iota m - n)u_{mn} = i(\mathcal{J} \nabla \cdot J_\perp)_{mn} = i \left[ \mathcal{J} \nabla \cdot \left( \frac{B \times \nabla p}{B^2} \right) \right]_{mn}$$
The parallel current appears to have a non-integrable $1/x$ radial singularity at flux surfaces for which $\nu$ is a rational number.

To avoid this, at rational surfaces

$$\frac{dp}{d\psi_t} = 0 \quad \text{or} \quad \left[ \mathcal{J} \nabla \cdot \left( \frac{\mathbf{B} \times \nabla \psi_t}{B^2} \right) \right]_{mn} = 0$$

Magnetic fields with $\left[ \mathcal{J} \nabla \cdot \left( \frac{\mathbf{B} \times \nabla \psi_t}{B^2} \right) \right]_{mn} \neq 0$ are unable to support a finite pressure gradient at the resonant rational surface.

Numerical equilibrium codes address the complications due to current sheets and in different ways, as we will later see.
PART IV: NUMERICAL SOLVERS FOR IDEAL MHD EQUILIBRIUM
THE WORKHORSE: VMEC

Two well-established equilibrium codes VMEC\(^2\) and NSTAB\(^3\) based on very similar principles

- Kruskal-Kulsrud energy variational principle\(^4\)
- Minimizes energy in space of equilibria with nested flux surfaces
  Assumes form of magnetic field with nested flux surfaces

\[
\mathbf{B} = \nabla \psi_t \times \nabla \theta + \nabla \psi_t \times \nabla \lambda - \nabla \psi_p(\psi_t) \times \nabla \varphi
\]

with \(\lambda(\psi_t, \theta, \varphi)\) periodic in \(\theta\) and \(\varphi\)

Flux conservation in energy principles guarantees that flux surfaces remain nested throughout minimization
The flux surfaces deform but do not break

- A large majority of numerical stellarator computations rely on equilibrium computed with VMEC


Equilibria with (imposed) nested surfaces
EQUILIBRIA WITHOUT IMPOSING NESTED SURFACES

The PIES\(^5\) code is designed to solve the MHD equilibrium equations without assuming the existence of nested flux surfaces

The MHD equations are solved iteratively. Each iteration consists in:

1. \( \mathbf{B} \cdot \nabla p = 0 \)

2. \( \mathbf{J}_\perp = \frac{\mathbf{B} \times \nabla p}{B^2} \)

3. \( \mathbf{B} \cdot \nabla \left( \frac{J_\parallel}{B} \right) = -\nabla \cdot \mathbf{J}_\perp \)

4. \( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \)

Eq.(1) is used to compute straight field line coordinates. This is where most of the action is – complications with rational surfaces and field line stochasticity

Eq.(2) and (3) can then be solved algebraically for each Fourier mode, where good surfaces exist.

Eq. (4) can finally be solved with standard solvers for Ampere’s law.

Equilibria without nested flux surfaces

PIES NCSX calculation\textsuperscript{6}  

- Convergence more challenging than with VMEC
- Run time substantially larger than VMEC
- PIES or HINT2 often take VMEC calculation as initial guess

HINT2 calculation of perturbed tokamak equilibrium\textsuperscript{7}

\textsuperscript{6}A. Reiman et al. Fusion Science and Technology \textbf{51}, 145 (2007)
\textsuperscript{7}Y. Suzuki, Plasma Physics and Controlled Fusion \textbf{59}, 054008 (2017)
TAYLOR RELAXATION

Any plasma has finite resistivity

Waiting long enough, the magnetic field will self-organize, breaking flux conservation but keeping the magnetic helicity conserved

Magnetic helicity: \( \mathcal{K} = \int_V \mathbf{A} \cdot \mathbf{B} dV \)

Taylor state: Minimize total energy subject to the constraint of conserved magnetic helicity

Mathematically, extremize

\[
\int_V \left( \frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) + \frac{\mu}{2} \left( \int_V \mathbf{A} \cdot \mathbf{B} dV - \mathcal{K}_0 \right)
\]

to obtain

\[ \nabla \times \mathbf{B} = \mu \mathbf{B} \quad \text{and} \quad p = \text{constant} \quad \text{in} \quad V \]
A half-way house between VMEC and PIES: MRxMHD and SPEC

Equilibrium described as nested Taylor states separated by ideal MHD interfaces at irrational values of $\iota$

In each volume, $\nabla \times \mathbf{B} = \mu_i \mathbf{B}$ and $p = \text{constant}$

Ideal MHD interfaces found iteratively, so that at each interface force balance holds:

$$\left[ \frac{B^2}{2\mu_0} + p \right] = 0 \quad \text{at each interface}$$

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(VERY BRIEF) PART V: MHD STABILITY
LINEAR STABILITY

- Ideal MHD dynamics can be so fast and detrimental that one may often require linear stability for the equilibrium

- Linearly perturb MHD equilibrium

- Two approaches:
  - Variational approach
    Compute second variation of potential energy $E$, historically (misleadingly) called $\delta W[\delta \xi]$
    If for all possible $\delta \xi$, $\delta W[\delta \xi] > 0$, equilibrium is linearly stable
  - Eigenmode analysis
    $-\rho \omega^2 \delta \xi = F(\delta \xi)$
    If all eigenmodes have $\omega^2 > 0$, equilibrium is linearly stable
COMMON MHD INSTABILITIES (I)

Kink instability

(Figure from Plasma Physics and Fusion Energy by J.P. Freidberg, CUP, 2008)
Common MHD instabilities (II)

Interchange instability

(Figure from *Plasma Physics and Fusion Energy* by J.P. Freidberg, CUP, 2008)
Common MHD instabilities (III)

Ballooning instability

(Figure from *Plasma Physics and Fusion Energy* by J.P. Freidberg, CUP, 2008)
Ballooning modes in tokamaks

(Left image from Hua-Sheng XIE’s website)
(Right image from
IDEAL MHD STABILITY - STILL QUITE OPEN

Unlike tokamaks, breaking stability limits does not lead to fast termination of stellarator discharges and dangerous release of energy.

Breaking stability limits can lead to degradation of discharge\textsuperscript{9,10} quality.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Graph showing low-n unstable and Te flattening phenomena.}
\end{figure}

LHD experiment

Incompletely understood phenomena. Ideal MHD linear stability less critical for stellarator design than for tokamaks?

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{w7x_equilibria.png}
\caption{W7-X equilibria.}
\end{figure}

\textsuperscript{9}Sakakibara, S. et al., Fusion Science and Technology 58, 176 (2010)
\textsuperscript{10}P. Helander et al., Plasma Physics and Controlled Fusion 54, 124009 (2012)