

# Coordinate Descriptions of Magnetic Fields

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**Clever coordinates  $(\xi^1, \xi^2, \xi^3)$  are required to understand magnetic fields that depend non-trivially on all three spatial coordinates.**

$$\vec{x}(\xi^1, \xi^2, \xi^3) = x(\xi^1, \xi^2, \xi^3)\hat{x} + y(\xi^1, \xi^2, \xi^3)\hat{y} + z(\xi^1, \xi^2, \xi^3)\hat{z}$$

$(x, y, z)$  are Cartesian coordinates and superscripts just number  $(\xi^1, \xi^2, \xi^3)$ .

Certainly true for understanding stellarators, but is also true more generally—whether in the laboratory or in space.

What is generally taught is orthogonal coordinates in three space,  $\frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j} = 0$  when  $i \neq j$ , and only part of that. What is needed is the general theory of coordinates in three dimensional space.

*A one-hour lecture is too short for mastery. For that read Boozer, Rev. Mod. Phys. 76, 1071 (2004), especially the appendix.*

## Derivation of Cylindrical Coordinates

$$\vec{x}(r, \theta, z) = x(r, \theta)\hat{x} + y(r, \theta)\hat{y} + z\hat{z}$$
$$x(r, \theta) = r \cos \theta \quad \text{and} \quad y(r, \theta) = r \sin \theta.$$

### Three tangent vectors

$$\frac{\partial \vec{x}}{\partial r} = \cos \theta \hat{x} + \sin \theta \hat{y}, \quad \frac{\partial \vec{x}}{\partial \theta} = -r \sin \theta \hat{x} + r \cos \theta \hat{y}, \quad \text{and} \quad \frac{\partial \vec{x}}{\partial z} = \hat{z}.$$

$$\text{Jacobian} \quad \mathcal{J} \equiv \left( \frac{\partial \vec{x}}{\partial r} \times \frac{\partial \vec{x}}{\partial \theta} \right) \cdot \frac{\partial \vec{x}}{\partial z} = r$$

### Three gradient vectors

Calculated using the dual relations, which are explained later, otherwise almost impossibly complicated.

$$\vec{\nabla} r = \frac{\frac{\partial \vec{x}}{\partial \theta} \times \frac{\partial \vec{x}}{\partial z}}{\mathcal{J}} = \cos \theta \hat{x} + \sin \theta \hat{y}, \quad \vec{\nabla} \theta = \frac{\frac{\partial \vec{x}}{\partial z} \times \frac{\partial \vec{x}}{\partial r}}{\mathcal{J}} = \frac{-\sin \theta \hat{x} + \cos \theta \hat{y}}{r}, \quad \text{and}$$
$$\vec{\nabla} z = \frac{\frac{\partial \vec{x}}{\partial r} \times \frac{\partial \vec{x}}{\partial \theta}}{\mathcal{J}} = \hat{z}. \quad \text{Note } (\vec{\nabla} r \times \vec{\nabla} \theta) \cdot \vec{\nabla} z = \frac{1}{\mathcal{J}}.$$

### Three crossed gradient vectors

$$\frac{\partial \vec{x}}{\partial r} = \mathcal{J} \vec{\nabla} \theta \times \vec{\nabla} z, \quad \frac{\partial \vec{x}}{\partial \theta} = \mathcal{J} \vec{\nabla} z \times \vec{\nabla} r, \quad \text{and} \quad \frac{\partial \vec{x}}{\partial z} = \mathcal{J} \vec{\nabla} r \times \vec{\nabla} \theta$$

# General Coordinates in Three Dimensional Space

Coordinates, such as  $(r, \theta, z)$  are denoted by superscripts  $(\xi^1, \xi^2, \xi^3)$ .

**The orthogonality theorem**  $\frac{\partial \vec{x}}{\partial \xi^i} \cdot \vec{\nabla} \xi^j = \delta_j^i$ , most important result.

$$\frac{\partial \vec{x}}{\partial \xi^i} \cdot \vec{\nabla} \xi^j = \frac{\partial x}{\partial \xi^i} \cdot \frac{\partial \xi^j}{\partial x} + \frac{\partial y}{\partial \xi^i} \cdot \frac{\partial \xi^j}{\partial y} + \frac{\partial z}{\partial \xi^i} \cdot \frac{\partial \xi^j}{\partial z} = \frac{\partial \xi^j}{\partial \xi^i} \text{ with } \xi\text{'s other than } \xi^i \text{ held constant.}$$

**The gradient:** 
$$\vec{\nabla} f(\xi^1, \xi^2, \xi^3) = \sum_i \frac{\partial f}{\partial \xi^i} \vec{\nabla} \xi^i$$

**Covariant vector:** 
$$\vec{A} = \sum_j A_j \vec{\nabla} \xi^j \text{ where } A_j = \vec{A} \cdot \frac{\partial \vec{x}}{\partial \xi^j}$$

$$\vec{\nabla} \times \vec{A} = \sum_{ij} \frac{\partial A_j}{\partial \xi^i} \vec{\nabla} \xi^i \times \vec{\nabla} \xi^j.$$

**Contravariant vector:** 
$$\vec{B} = \sum_i B^i \frac{\partial \vec{x}}{\partial \xi^i} \text{ where } B^i = \vec{B} \cdot \vec{\nabla} \xi^i.$$

$$\frac{\partial \vec{x}}{\partial \xi^1} = \mathcal{J} \vec{\nabla} \xi^2 \times \vec{\nabla} \xi^3, \quad \vec{\nabla} \cdot \left( B^1 \frac{\partial \vec{x}}{\partial \xi^1} \right) = \frac{1}{\mathcal{J}} \frac{\partial}{\partial \xi^1} (\mathcal{J} B^1), \text{ where } \frac{1}{\mathcal{J}} = \vec{\nabla} \xi^1 \cdot (\vec{\nabla} \xi^2 \times \vec{\nabla} \xi^3).$$

# General Contravariant Magnetic Field Representation

Toroidal  $(r, \theta, \varphi)$  Cartesian coordinate relations

$$x = (R_0 + r \cos \theta) \cos \varphi$$

$$y = (R_0 + r \cos \theta) \sin \varphi$$

$$z = -r \sin \theta$$

The vector potential of the magnetic field is

$$\vec{A} = A_r \vec{\nabla} r + A_\theta \vec{\nabla} \theta + A_\varphi \vec{\nabla} \varphi + \vec{\nabla} g, \quad \text{where} \quad \vec{B} = \vec{\nabla} \times \vec{A}.$$

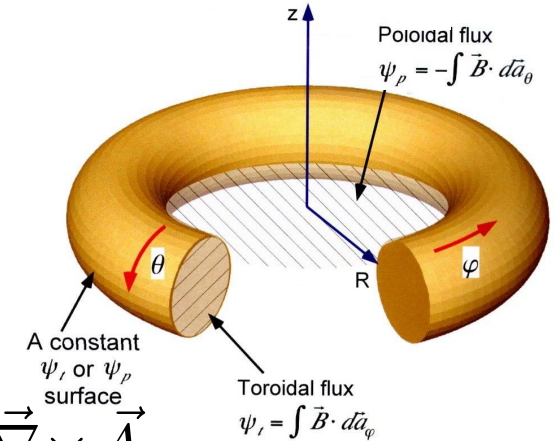
Let  $\partial g / \partial r = -A_r$ ,  $\psi_t / 2\pi \equiv A_\theta + \partial g / \partial \theta$ , and  $\psi_p / 2\pi \equiv -A_\varphi - \partial g / \partial \varphi$ , then

$$\vec{B} = \vec{\nabla} \psi_t \times \vec{\nabla} \frac{\theta}{2\pi} + \vec{\nabla} \frac{\varphi}{2\pi} \times \vec{\nabla} \psi_p \quad \text{general contravariant representation.}$$

Toroidal field is non zero in the plasma,  $\vec{B} \cdot \vec{\nabla} \varphi \neq 0$ , so  $(\vec{\nabla} \psi_t \times \vec{\nabla} \theta) \cdot \vec{\nabla} \varphi \neq 0$ .

**When  $\vec{B}$  is time dependent,  $\psi_p(\psi_t, \theta, \varphi, t)$  and  $\vec{x}(\psi_t, \theta, \varphi, t)$ ; lines of  $\vec{B}$  cannot change topology when  $\psi_p$  is independent of time.**

$\vec{x}(\psi_t, \theta, \varphi, t)$  is a homotopy;  $g_{ij} \equiv \frac{\partial \vec{x}}{\partial \xi^i} \cdot \frac{\partial \vec{x}}{\partial \xi^j}$  is the metric tensor.

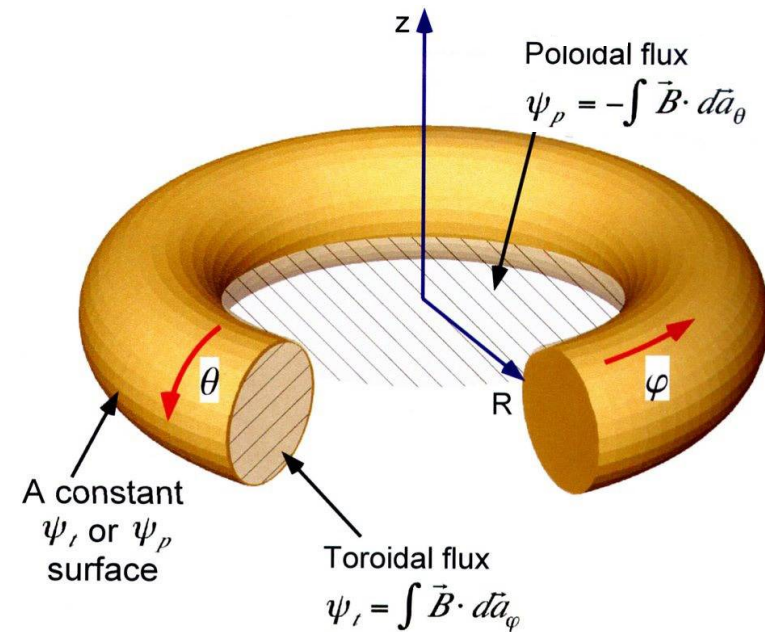


## Magnetic Field Lines $\frac{d\vec{x}}{d\tau} = \vec{B}$

$$\vec{B} = \frac{d\vec{x}}{d\tau} = \sum_i \frac{\partial \vec{x}}{\partial \xi^i} \frac{d\xi^i}{d\tau}, \quad \text{so} \quad \frac{d\xi^i}{d\tau} = \vec{B} \cdot \vec{\nabla}_{\xi^i}$$

## Magnetic Field Line Hamiltonian $\psi_p(\psi_t, \theta, \varphi)$

$$\begin{aligned} \frac{d\psi_t}{d\varphi} &\equiv \frac{\vec{B} \cdot \vec{\nabla} \psi_t}{\vec{B} \cdot \vec{\nabla} \varphi} = - \frac{\partial \psi_p(\psi_t, \theta, \varphi)}{\partial \theta} \\ \frac{d\theta}{d\varphi} &\equiv \frac{\vec{B} \cdot \vec{\nabla} \theta}{\vec{B} \cdot \vec{\nabla} \varphi} = + \frac{\partial \psi_p(\psi_t, \theta, \varphi)}{\partial \psi_t}. \end{aligned}$$



*Standard Hamiltonian mechanics is*

$$\frac{dp}{dt} = - \frac{\partial H(p, q, t)}{\partial q} \quad \text{and} \quad \frac{dq}{dt} = \frac{\partial H}{\partial p}.$$

## Magnetic Surfaces, $\vec{B} \cdot \vec{\nabla} f(\vec{x}) = 0$ , and Magnetic Coordinates

When magnetic surfaces exist, the contravariant form for  $\vec{B}$  is simplified.

$$\vec{B} = \frac{B_1}{2\pi} \vec{\nabla} f \times \vec{\nabla} \theta + \frac{B_2}{2\pi} \vec{\nabla} \varphi \times \vec{\nabla} f \text{ with } \vec{\nabla} \cdot \vec{B} = 0 \text{ implying } \frac{\partial B_1}{\partial \varphi} + \frac{\partial B_2}{\partial \theta} = 0.$$

The general solution uses  $\Psi_t(f)$ ,  $\Psi_p(f)$  and  $\lambda(f, \theta, \varphi)$ ,

$$B_1 = \left(1 + \frac{\partial \lambda}{\partial \theta}\right) \frac{d\Psi_t}{df} \text{ and } B_2 = \frac{d\Psi_p}{df} - \frac{\partial \lambda}{\partial \varphi} \frac{d\Psi_t}{df}.$$

Let  $\vartheta \equiv \theta + \lambda$ , then  $(\Psi_t, \vartheta, \varphi)$  are magnetic coordinates;

$$\vec{B} = \vec{\nabla} \Psi_t \times \vec{\nabla} \frac{\vartheta}{2\pi} + \vec{\nabla} \frac{\varphi}{2\pi} \times \vec{\nabla} \Psi_p(\Psi_t).$$

The rotational transform is  $\iota \equiv \frac{d\Psi_p}{d\Psi_t}$  and  $\vartheta = \Theta + \iota\varphi$ , where  $\Theta$  is a field-line constant;  $\vec{B} = \vec{\nabla} \psi_t \times \vec{\nabla} \frac{\Theta}{2\pi}$  is the Clebsch representation.

**Normal notation for magnetic coordinates is  $(\psi_t, \theta, \varphi)$ .**

## Magnetic Differential Equation $\vec{B} \cdot \vec{\nabla} f = g$

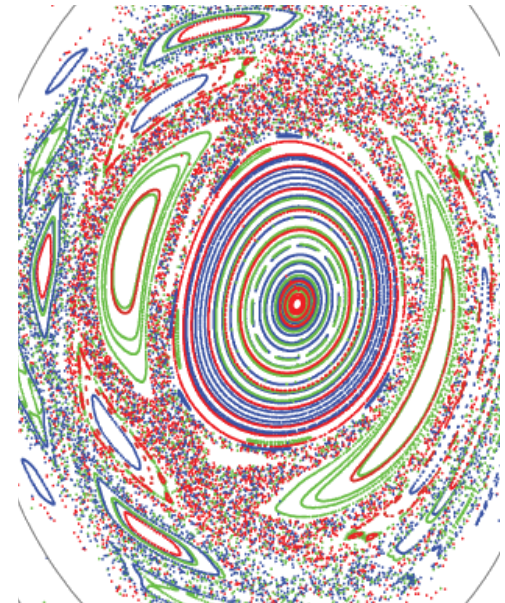
Solutions essentially require magnetic coordinates, then  $\left( \frac{\partial}{\partial \varphi} + \iota \frac{\partial}{\partial \theta} \right) f = \frac{g}{\vec{B} \cdot \vec{\nabla} \varphi}$ .

Write  $\frac{g}{\vec{B} \cdot \vec{\nabla} \varphi} = \sum_{mn} \gamma_{mn} \sin(n\varphi - m\theta)$ , then  $f = f_0(\psi_t) - \sum_{mn} \frac{\gamma_{mn}}{n - \iota m} \cos(n\varphi - m\theta)$ .

The solution is singular on rational surfaces  $\iota(\psi_{MN}) = N/M$ , with  $M$  and  $N$  integers, which causes magnetic islands and stochastic regions.

A simple calculation can be made of islands. The magnetic field line Hamiltonian allows a complete picture.

*It is simpler to obtain an intuitive understanding of Hamiltonian mechanics by thinking about magnetic field lines than the other way around.*



## Magnetic Islands

When a magnetic field  $\vec{B}_0$  with perfect surfaces is perturbed by a field  $\delta\vec{B}$ , the perturbed surfaces are given by  $f = f_0(\psi_t) + \delta f = \text{const.}$ , where  $(\vec{B}_0 + \delta\vec{B}) \cdot \vec{\nabla} f = 0$ , so

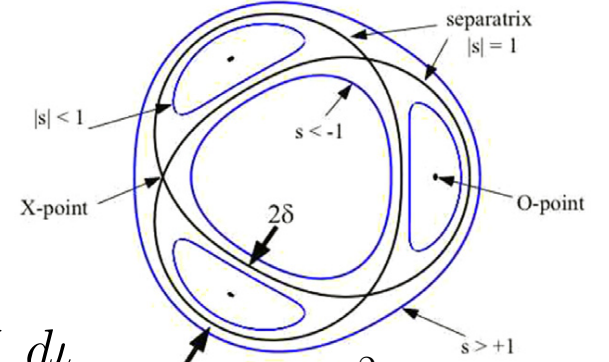
$$\vec{B}_0 \cdot \vec{\nabla} \delta f = -\delta\vec{B} \cdot \vec{\nabla} \psi_t \frac{df_0}{d\psi_t}.$$

Let  $\frac{\delta\vec{B} \cdot \vec{\nabla} \psi_t}{\vec{B}_0 \cdot \vec{\nabla} \varphi} = b_{MN} \sin(N\varphi - M\theta)$ , and  $f_0 = \frac{M}{2} \frac{d\iota}{d\psi_t} (\psi_t - \psi_{MN})^2$ .

$$\delta f = b_{MN} \cos(N\varphi - M\theta) = b_{MN} \left( 1 - 2 \sin^2 \left( \frac{N\varphi - M\theta}{2} \right) \right) \quad \text{so}$$

$$\psi_t = \psi_{mn} + \frac{s}{|s|} \sqrt{\frac{4b_{MN}}{md\iota/d\psi_t} \left( s^2 - \sin^2 \left( \frac{N\varphi - M\theta}{2} \right) \right)},$$

where  $s^2 \equiv f + b_{mn} \geq 0$  is a constant;  $s^2 > 1$  means outside and  $s^2 < 1$  is inside the island.



The island halfwidth is  $\delta \equiv \sqrt{\frac{4b_{MN}}{md\iota/d\psi_t}}.$



## Covariant Representation for $\vec{B}$ when $\vec{\nabla} p(\psi_t) = \vec{j} \times \vec{B}$

Since  $\vec{j} \cdot \vec{\nabla} \psi_t = 0$  and  $\vec{\nabla} \cdot \vec{j} = 0$ , the current density has the representation

$$\vec{j} = -\frac{\partial \mathcal{G}}{\partial \psi_t} \frac{\vec{\nabla} \varphi \times \vec{\nabla} \psi_t}{2\pi} + \frac{\partial \mathcal{I}}{\partial \psi_t} \frac{\vec{\nabla} \psi_t \times \vec{\nabla} \theta}{2\pi}, \quad \text{where} \quad \frac{\partial}{\partial \theta} \frac{\partial \mathcal{G}}{\partial \psi_t} = \frac{\partial}{\partial \varphi} \frac{\partial \mathcal{I}}{\partial \psi_t}.$$

$$\text{Consequently, } \frac{\partial \mathcal{G}}{\partial \psi_t} = \frac{dG(\psi_t)}{d\psi_t} + \frac{\partial \nu}{\partial \varphi} \quad \text{and} \quad \frac{\partial \mathcal{I}}{\partial \psi_t} = \frac{dI(\psi_t)}{d\psi_t} + \frac{\partial \nu}{\partial \theta}.$$

$$\text{Since } \vec{\nabla} \times \vec{B} = \mu_0 \vec{j}, \text{ the field } \vec{B} = \frac{\mu_0}{2\pi} \left\{ G(\psi_t) \vec{\nabla} \varphi + I(\psi_t) \vec{\nabla} \theta - \nu \vec{\nabla} \psi_t + \vec{\nabla} F \right\}$$

The contravariant representation of  $2\pi \vec{B} = \vec{\nabla} \psi_t \times \vec{\nabla} \theta + \iota \vec{\nabla} \varphi \times \vec{\nabla} \psi_t$  is unchanged by new angles  $\theta = \theta_n + \iota \omega$  and  $\varphi = \varphi_n + \omega$ , so the covariant representation can be written

$$\vec{B} = \frac{\mu_0}{2\pi} \left\{ G \vec{\nabla} \varphi_n + I \vec{\nabla} \theta_n - \nu_n \vec{\nabla} \psi_t + \vec{\nabla} F_n \right\}; \quad \nu_n = \nu + \left( \frac{dG}{d\psi_t} + \iota \frac{dI}{d\psi_t} \right) \omega; \quad F_n = F + (G + \iota I) \omega.$$

$$\text{Hamada (1962) chose } \nu_n = 0 \text{ giving} \quad \vec{B} = \frac{\mu_0}{2\pi} \left\{ G \vec{\nabla} \varphi_H + I \vec{\nabla} \theta_H + \vec{\nabla} F_H \right\}$$

$$\text{Boozer (1981) chose } F_n = 0 \text{ giving} \quad \vec{B} = \frac{\mu_0}{2\pi} \left\{ G \vec{\nabla} \varphi_B + I \vec{\nabla} \theta_B \right\} + \beta_* \vec{\nabla} \psi_t.$$

## Particle Drift Hamiltonian

The lowest order Hamiltonian for the particle drift motion is easily obtained using Boozer coordinates.

$$H = \frac{1}{2}mv_{\parallel}^2 + \mu B + q\Phi \quad \text{where the adiabatic invariant} \quad \mu = \frac{mv_{\perp}^2}{2B}$$

The particle mass and charge are  $m$  and  $q$ . The velocity of the particle parallel to  $\vec{B}$  is  $\vec{v}_{\parallel}$ , and the velocity perpendicular to  $\vec{B}$  is  $\vec{v}_{\perp}$ . The electric potential is  $\Phi$ .

The momentum that is canonically conjugate to the position vector is  $\vec{P} = m\vec{v} + q\vec{A}$ , where  $\vec{A}$  is the vector potential;  $\vec{B} = \vec{\nabla} \times \vec{A}$ .

$$P_{\theta} \equiv \vec{P} \cdot \frac{\partial \vec{x}}{\partial \theta} \quad \text{and} \quad P_{\varphi} \equiv \vec{P} \cdot \frac{\partial \vec{x}}{\partial \varphi} \quad \text{are canonically conjugate to } \theta \text{ and } \varphi.$$

When the gyroradius is small, only the parallel component of the velocity remains large  $\vec{v} \rightarrow v_{\parallel} \vec{B}/B$  as the gyroradius to system size goes to zero.

In Boozer coordinates,  $\vec{B} \cdot \partial \vec{x} / \partial \theta = \mu_0 I / 2\pi$  and  $\vec{B} \cdot \partial \vec{x} / \partial \varphi = \mu_0 G / 2\pi$  and  $\vec{A} \cdot \partial \vec{x} / \partial \theta = \psi_t$  and  $\vec{A} \cdot \partial \vec{x} / \partial \varphi = -\psi_p$ :

$$P_{\theta} = \frac{mu_0 I(\psi_t)}{2\pi B} m v_{\parallel} + q \psi_t \quad \text{and} \quad P_{\varphi} = \frac{mu_0 G(\psi(t))}{2\pi B} m v_{\parallel} - q \psi_p.$$

# Fundamental Results

## General Contravariant Representation of $\vec{B}$ :

$$\vec{B} = \vec{\nabla}\psi_t \times \vec{\nabla}\frac{\theta}{2\pi} + \vec{\nabla}\frac{\varphi}{2\pi} \times \vec{\nabla}\psi_p.$$

## Magnetic Field Line Hamiltonian:

$$\frac{d\psi_t}{d\varphi} = -\frac{\partial\psi_p(\psi_t, \theta, \varphi)}{\partial\theta}; \quad \frac{d\theta}{d\varphi} = +\frac{\partial\psi_p(\psi_t, \theta, \varphi)}{\partial\psi_t}.$$

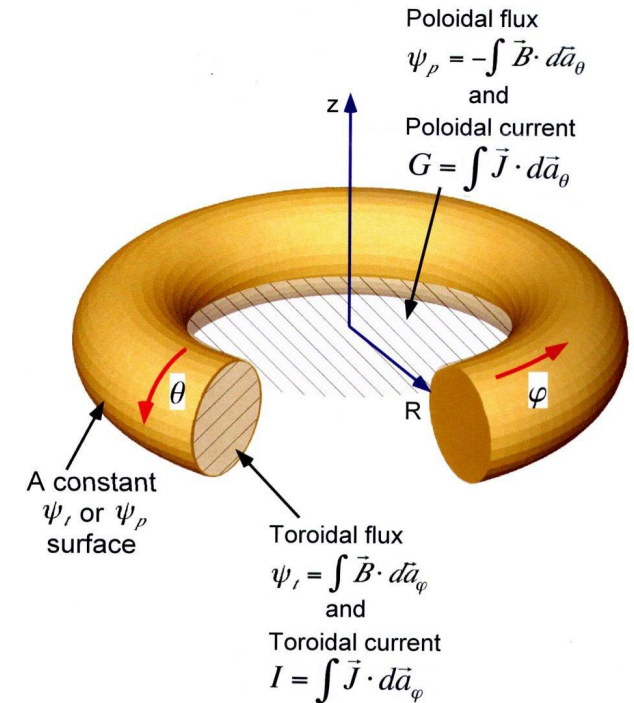
## Covariant Representation of $\vec{B}$ :

$$\vec{B} = \frac{\mu_0 G(\psi_t)}{2\pi} \vec{\nabla}\theta + \frac{\mu_0 I(\psi_t)}{2\pi} \vec{\nabla}\varphi + \beta_*(\psi_t, \theta, \varphi) \vec{\nabla}\psi_t$$

## Particle Drift Hamiltonian: $H(P_\theta, \theta, P_\varphi, \varphi) = \frac{m}{2}v_{||}^2 + \mu B + q\Phi;$

$$P_\theta = \frac{\mu_0 I}{2\pi B} m v_{||} + \frac{q}{2\pi} \psi_t; \quad P_\varphi = \frac{\mu_0 G}{2\pi B} m v_{||} - \frac{q}{2\pi} \psi_p.$$

When  $\theta = \vartheta + N\varphi$  and  $B(\psi_t, \vartheta)$ , then  $P_c \equiv P_\varphi - NP_\theta$  is conserved—called quasi-symmetry.



## Inventions in Stellarator Design

Lectures naturally focus on what is known—not where major innovations and inventions are possible.

Lectures on stellarator design should be different. Much has been achieved, but obvious opportunities for major advancements remain for anyone willing to explore them.

*The stellarator is unique among all fusion concepts, inertial and magnetic, in not requiring any part of the state in which the plasma is confined to be produced by the plasma itself which makes computational design uniquely reliable.*

- The cost of computational design is between 0.1% and 1% of the cost of building a major experiment.
- Experiments are costly, (a) build in conservatism—even apparently minor changes in design are not possible and therefore remain unstudied—and (b) are built and operated over long periods time (decades).
- Ideally experiments should only be built to validate a computational design.

## Coil Design

1. Can coils be designed that allow easy access to the plasma chamber? Rapid changes in the structures surrounding the plasma are critical for rapid development of fusion energy.

*A green helical coil is shown wrapped around the chamber of a quasi-axisymmetric stellarator. The remainder of the external field could be produced by coils that do not encircle the plasma and are easily removed together with large wall segments. **Unexplored***



2. Can great improvements be made in coil design by using only those external magnetic field distributions that can be efficiently produced at a great distance?

*Curl-free magnetic fields decay through space as  $e^{-kx}$  where  $k$  is the wavenumber of the field. One can determine all possible external magnetic field and order them by their efficiency of production. **Benefits largely unexplored***

## Identification of Attractive Magnetic Fields

**Optimization:** Numerical optimizations can (1) refine an initial guess or (2) maintain the optimization of a curl-free magnetic field as the plasma pressure is increased.

*About fifty magnetic field distributions can be produced by coils with adequate efficiency for fusion applications—too many possibilities to explore them all.*

**Success requires identification of desirable starting points.**

**Annular Design:** An optimal stellarator would have low plasma transport in the outer quarter of the radius and rapid transport in the inner three quarters.

**Implications almost unexplored**

## A New Method of Identifying Attractive Magnetic Fields

Dot  $\frac{\partial \vec{x}}{\partial \theta}$  and  $\frac{\partial \vec{x}}{\partial \varphi}$  with the co- and contra-variant forms of  $\vec{B}$ ,

$$2\pi \vec{B} = \frac{G}{\mu_0} \vec{\nabla} \varphi + \frac{I}{\mu_0} \vec{\nabla} \theta + \beta_* \vec{\nabla} \psi_t = \frac{1}{\mathcal{J}} \left( \frac{\partial \vec{x}}{\partial \varphi} + \iota(\psi_t) \frac{\partial \vec{x}}{\partial \theta} \right),$$

to determine an outer magnetic surface with desirable properties.

**There are three free functions to satisfy two constraints.**

*On a  $\psi_t$  surface  $(R, \zeta, Z)$  cylindrical coordinates are related to  $(\theta, \varphi)$  magnetic coordinates by three periodic functions:*

$$R(\theta, \varphi), \quad \zeta = \varphi + \omega(\theta, \varphi), \quad \text{and } Z(\theta, \varphi).$$

**For curl-free solutions, the plasma current  $I = 0$ .**

The magnetic field everywhere can be determined by choosing the efficient distributions of external magnetic fields so there is no magnetic field normal to the surface  $\vec{x}(\theta, \varphi)$

**Plasma pressure doesn't change the external field when  $I = 0$ .**

*Method unexplored*

## Protection of the Walls from $\alpha$ Particle Damage

*Helium ions ( $\alpha$  particles) produced by the nuclear reactions can become deeply embedded in the walls if they strike while they are still energetic. The accumulation of helium gas in crystal lattices creates blisters and fuzzy regions, which destroys the structural integrity of the walls.*

Energetic  $\alpha$ 's that leave fusion plasmas are trapped particles executing banana orbits and in principle their trajectories could be controlled so they harmlessly strike a liquid, such as lithium or tin, not a solid wall.

***Feasibility essentially unexplored***



## Divertors to Carry Escaping Plasma to Pumps

*The particle exhaust from plasmas must be concentrated to the location of pumps but this concentration can make the power loading on the walls intolerably high unless a large fraction of the power can be radiated away.*

Two solutions:

**(1) Resonant divertor locates a chain of islands at the plasma edge.**

*This requires extremely accurate control of the edge transform,  $\iota$ .*

**(2) Non-resonant divertor uses the Hamiltonian mechanics concepts of Cantori and turnstiles.**

*Beyond the outermost confining magnetic surface, a double magnetic flux tube is formed (1/2 the flux comes in and 1/2 goes out), which strikes the wall at a remarkably robust location.*

***W7-X has a resonant divertor; non-resonant divertors relatively unexplored.***