Basic Plasma Physics for Fusion

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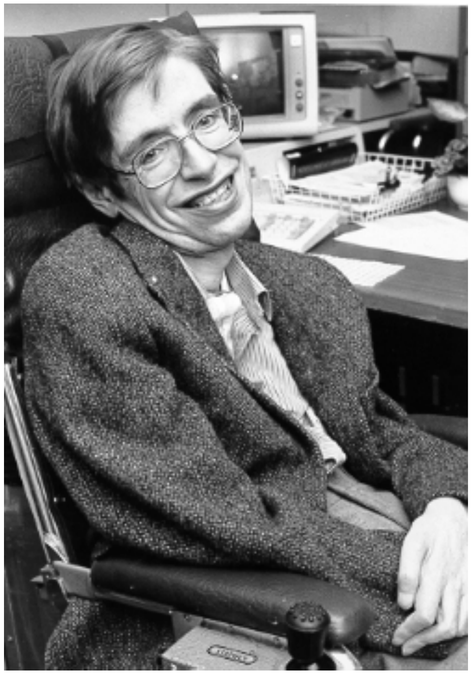
Quest for thermonuclear fusion motivates the search for optimum magnetic fields

"I would like nuclear fusion to become a practical power source. It would provide an inexhaustible supply of energy, without pollution or global warming."

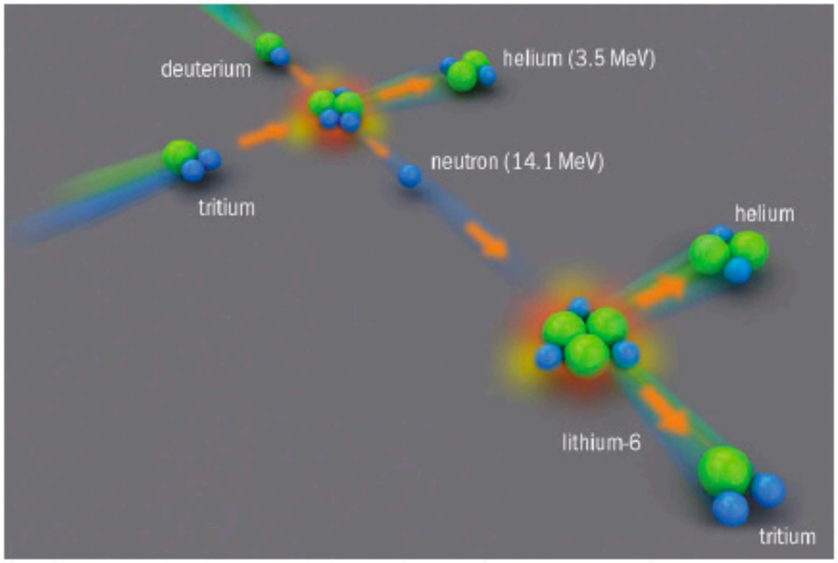
Stephen Hawking

(10 Questions for Stephen Hawking, Time [2010])



1942-2018

Thermonuclear fusion

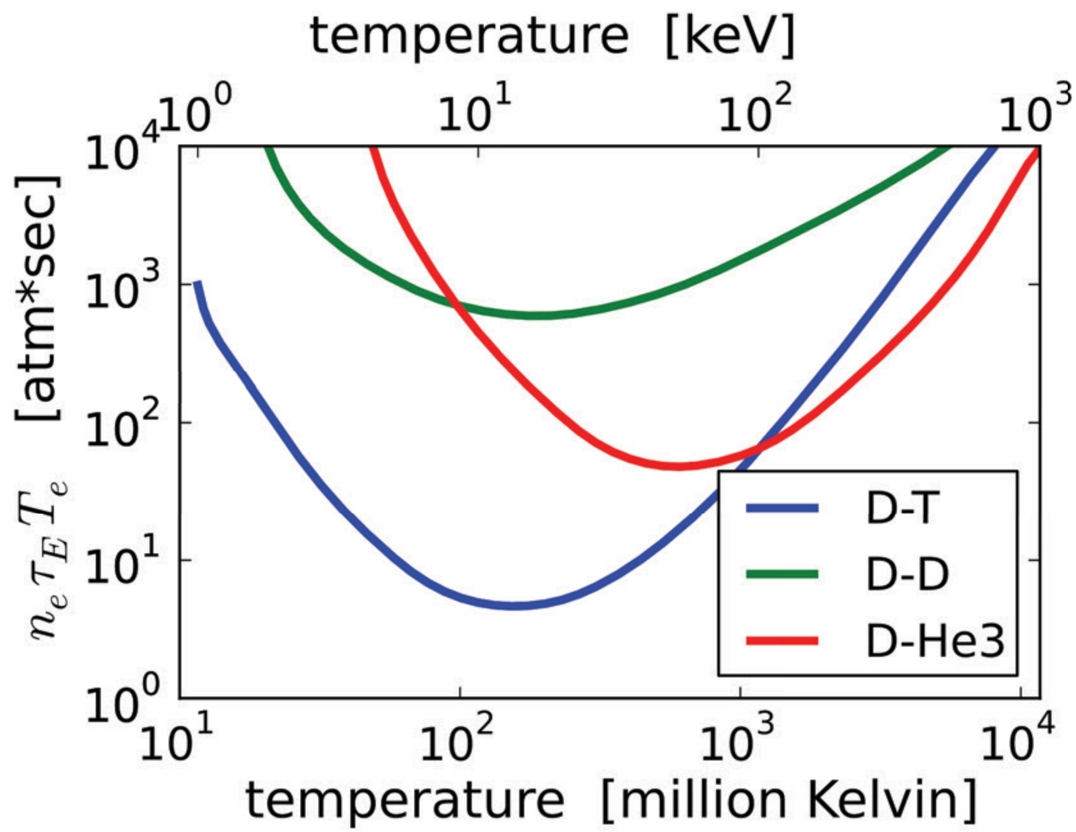


http://environmentalresearchweb.org/cws/article/opinion/43980

- Deuterium (1p, 1n) collides with Tritium (1p, 2n)
- After fusion, 17.6 *MeV* are released
- Released neutron splits lithium-6 into more tritium

Lawson's Criterion

$$nT\tau_E > 3 \times 10^{21}\ \mathrm{m^{-3}keVs}$$



What is a plasma?

Plasma is an ensemble of charged particles, capable of exhibiting collective interactions.

Levels of Description:

- Single-particle dynamics in prescribed electric and magnetic fields (T. Pedersen lecture)
- Plasmas as fluids in 3D configuration space moving under the influence of self-consistent electric and magnetic fields (A. Cerfon lecture)
- Plasmas as kinetic fluids in 6D phase space (that is, configuration and velocity space), coupled to self-consistent Maxwell's equations.

What does div B = 0 mean?

- Magnetic fields obey div **B** = 0. Traditional electromagnetic theory texts generally emphasize that this means that there are no isolated magnetic monopoles, so field lines either close on themselves, or go from infinity to infinity.
- But there is a third possibility: field lines can fill a compact surface or volume ergodically.
- In the presence of a direction of continuous symmetry, field lines fill surfaces ergodically. These surfaces are called *magnetic surfaces*.

Remark: Magnetic field line systems are Hamiltonian, and div **B** = 0 is their Liouville's theorem. When a Hamiltonian has an ignorable (or cyclic coordinate), it has a constant of the motion. For **B** fields, these are the magnetic surfaces. (More in A. Boozer lecture)

Magnetic Surfaces and Chaos

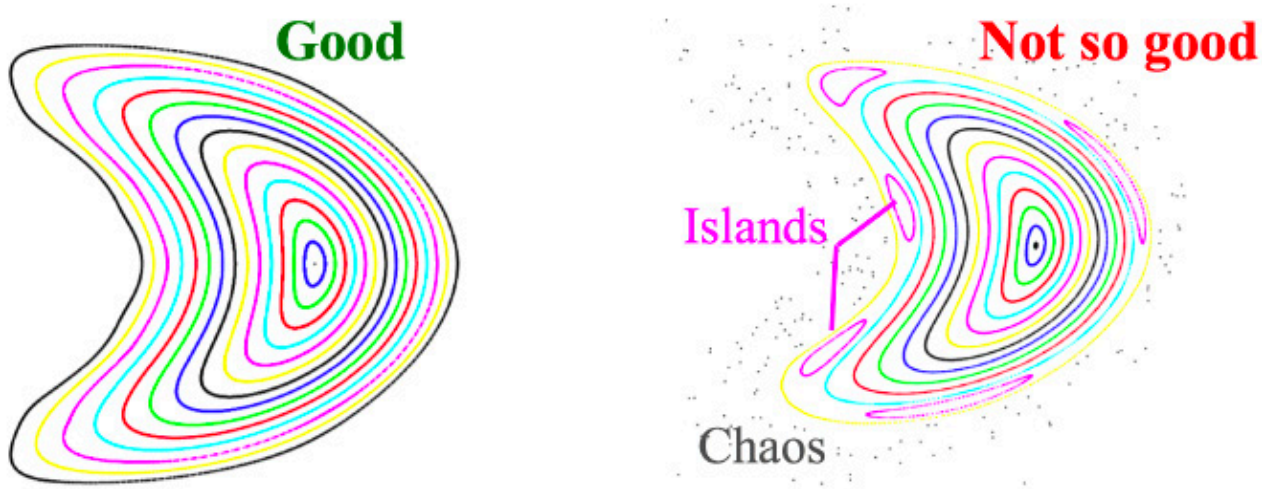
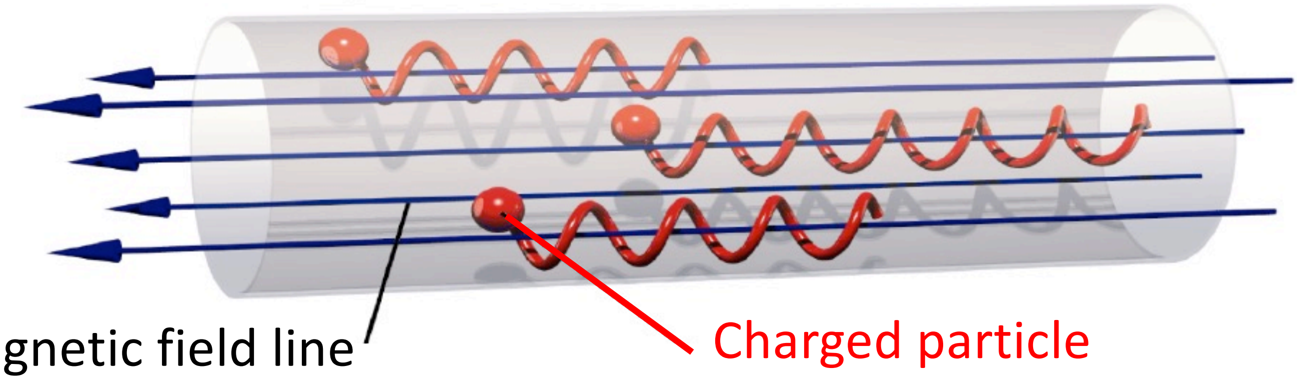


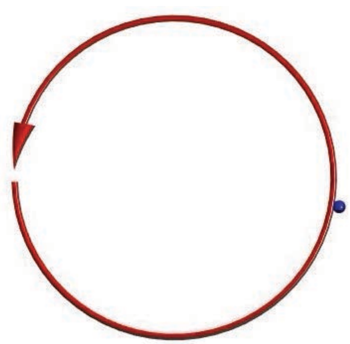
Figure 3: To produce this figure, field lines are followed around the device, and each time they hit a plane at constant toroidal angle, a point is marked on the plot with colors indicating a given field line. This is often referred to as a Poincaré plot. On the left, the field lines form closed, nested flux surfaces. On the right there is a region of 'good flux surfaces' along with a chain of island structures and chaotic field lines. Figure reproduced from [8].

From primer by Imbert-Gerard, Paul and Wright

Confining charged particles with a magnetic field is not as easy as it may seem

Uniform straight **B**: confinement ⊥ to **B**, but end losses can be very large





Axisymmetry is one way to achieve magnetic confinement

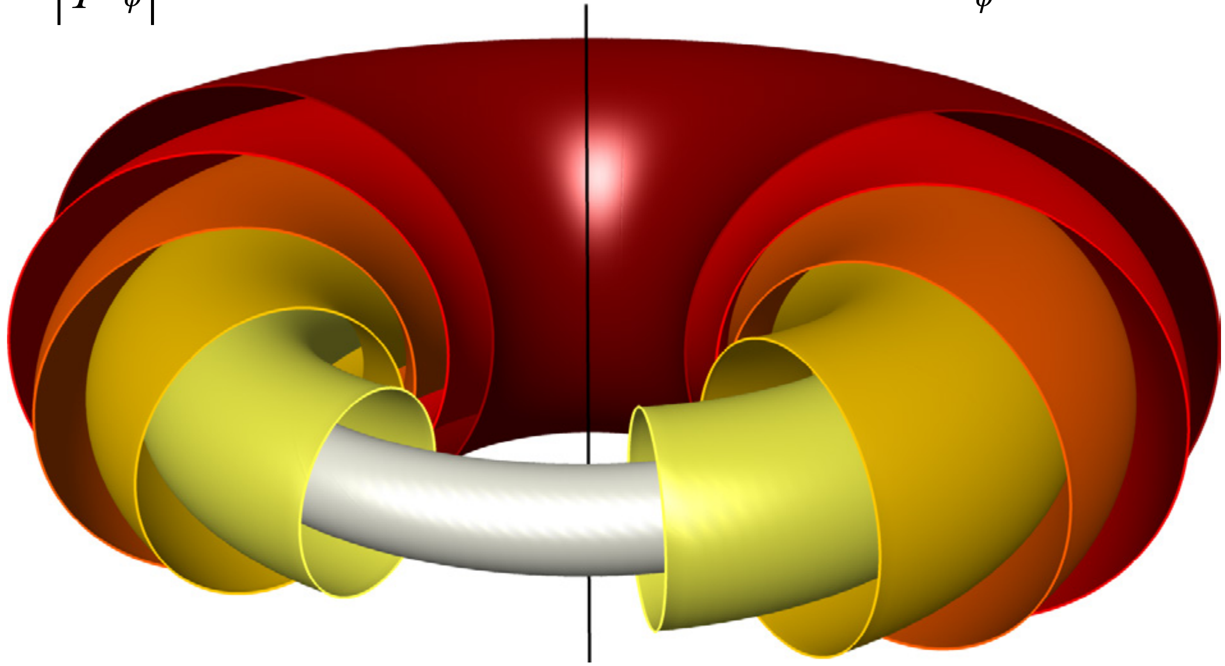
Continuous rotational symmetry $\Rightarrow$ Canonical angular momentum is conserved.

$$L_\phi = mv_\phi R + qA_\phi R = \text{constant}$$

$\nwarrow$ vector potential: $\mathbf{B} = \nabla \times \mathbf{A}$

Strong **B** limit $\Rightarrow |mv_\phi| \ll |qA_\phi| \Rightarrow$ Particles stuck to constant-$A_\phi R$ surfaces.

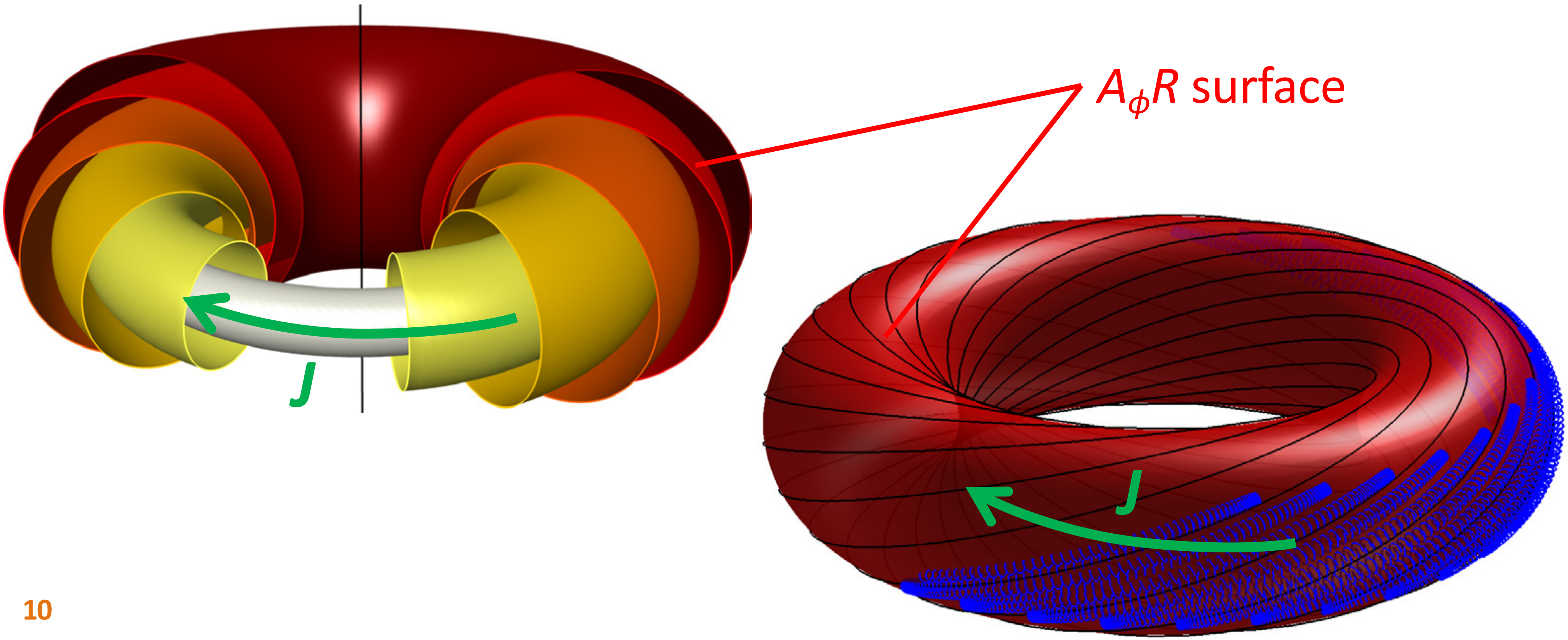
If $A_\phi R$ surfaces are bounded, then particles will be confined. This requires superposition of poloidal and toroidal fields which produce a rotational transform.



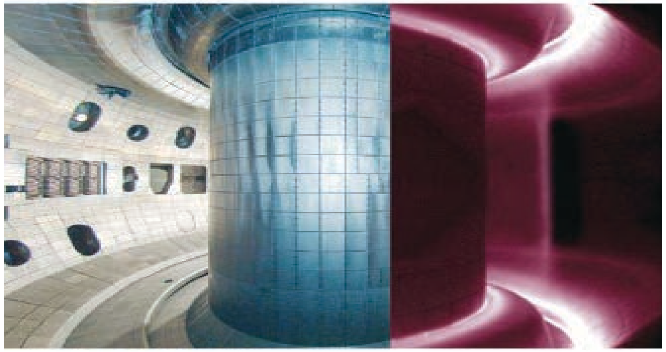
Complication: Axisymmetric confinement requires an internal current

$\nabla \times \left(\nabla \times \mathbf{A} \right) = \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ so nested $A_\phi R$ surfaces require a J_ϕ,

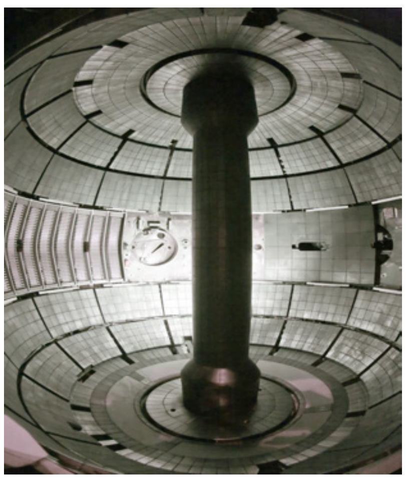
making the axisymmetric tokamak prone to instabilities/disruptions.



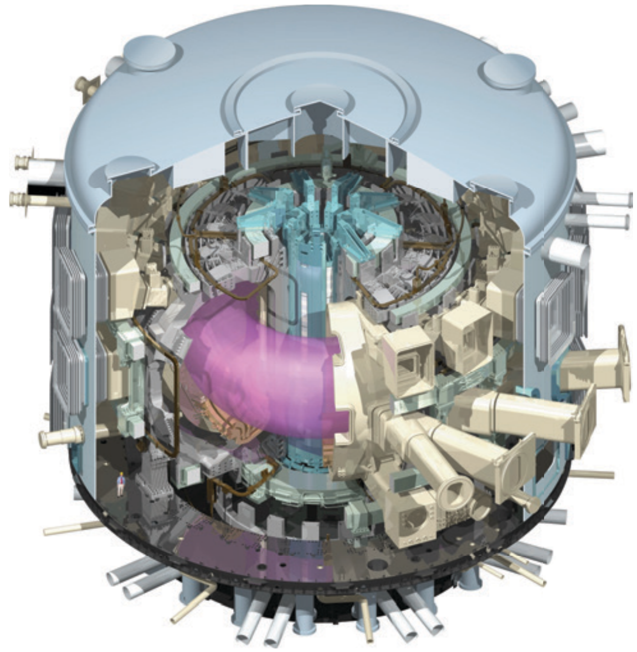
Fusion in a Tokamak: Some Examples



DIII-D , General Atomics, San Diego



NSTX-U, PPPL



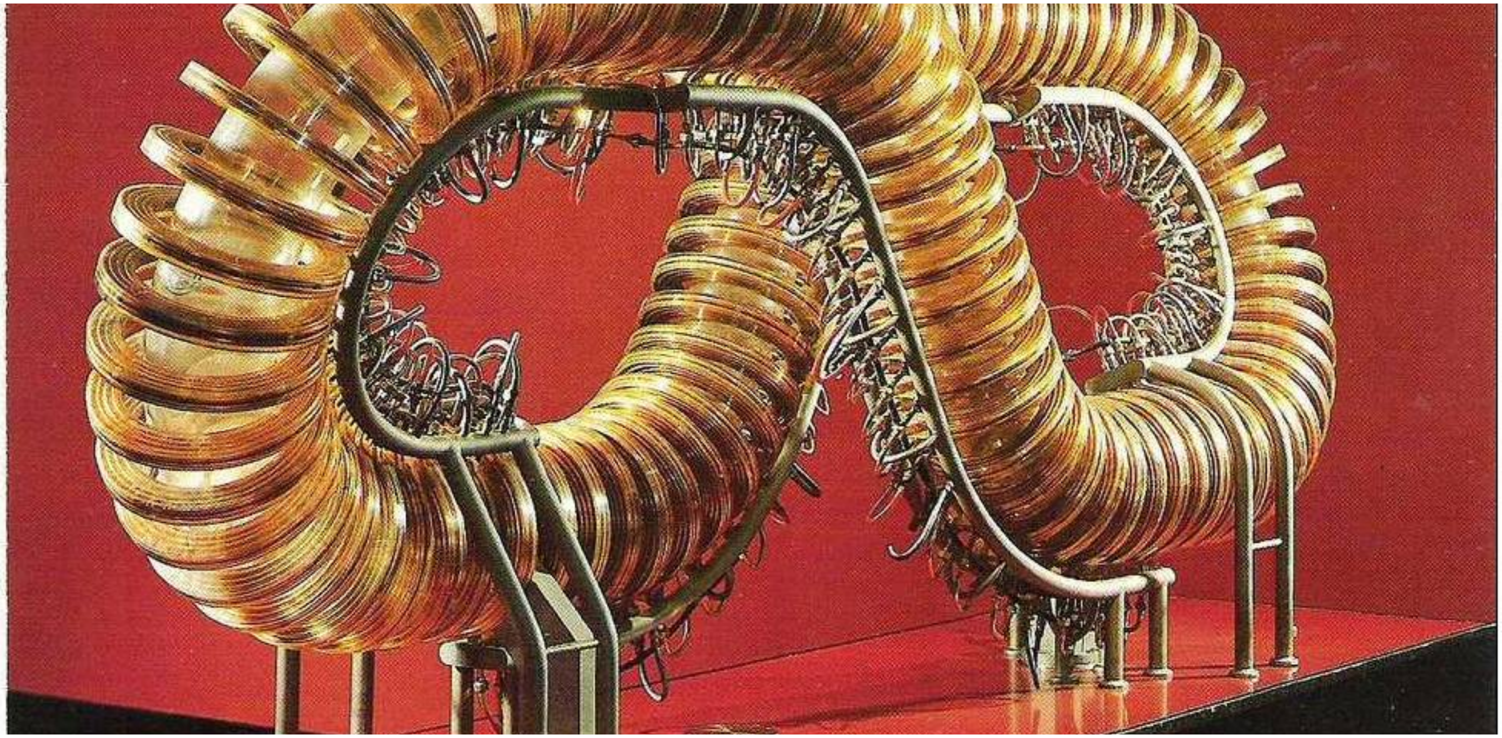
Target of ITER:
Q =10
Input 50MW,
Output 500MW

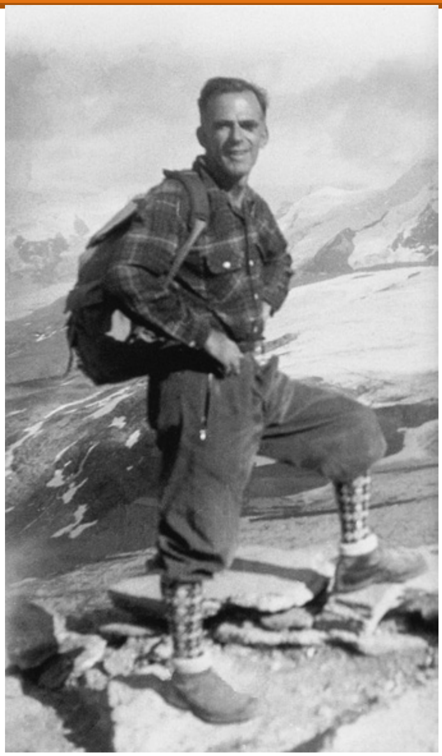
Challenges faced by ITER:

1. Plasma current source of disruptive instabilities that can produce highly energetic runaway electrons, which can damage confinement vessel.

2. Steady-state operation will require strong current drive.

Figure-8 Stellarator





Lyman Spitzer (1914-1997)

Poloidal magnetic field need not require an axial current, but can be purely an outcome of geometry, due to the torsion of magnetic field lines as in a Figure-8 stellarator (which anticipated the Berry phase in quantum mechanics).

Geometry is magic

- $\hat{e}_1^P = \hat{\boldsymbol{b}}(P)$, the unit tangent vector in the direction of the magnetic field;
- $\hat{e}_2^P = \frac{\boldsymbol{\kappa}(P)}{|\boldsymbol{\kappa}(P)|}$, the unit vector in the direction of the magnetic curvature $\boldsymbol{\kappa}(P) = (\hat{\boldsymbol{b}}(P) \cdot \nabla)\hat{\boldsymbol{b}}(P)$;
- $\hat{e}_3^P = \hat{e}_1^P \times \hat{e}_2^P$.

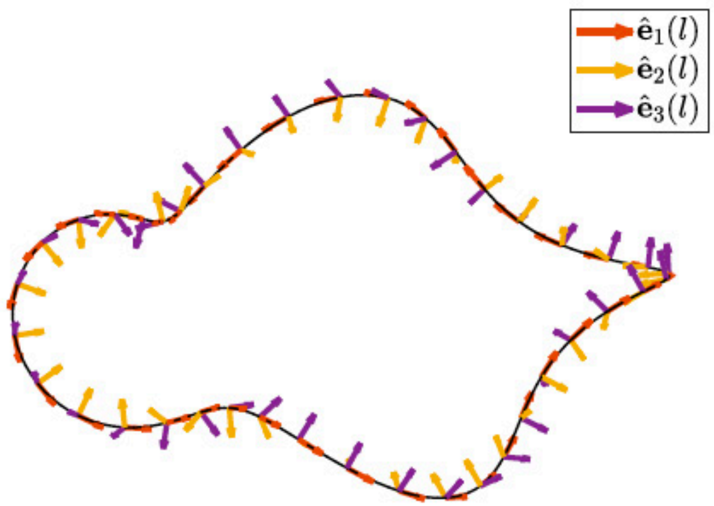


Figure 20: The magnetic axis of the TJ-II stellarator (black) is displayed with the orthonormal Frenet-Serret unit vectors.

From primer by Imbert-Gerard, Paul and Wright

Ideal MHD equations and magnetostatic equilibria

	$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$	
PDE	$\rho \left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla \right) \boldsymbol{u} = \boldsymbol{J} \times \boldsymbol{B} - \nabla p$	$\boldsymbol{J} \times \boldsymbol{B} = \nabla p$
model	$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla \right) \left(\frac{p}{\rho^\gamma} \right) = 0$	
	$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$	$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$
	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	
	$\nabla \cdot \boldsymbol{B} = 0$	$\nabla \cdot \boldsymbol{B} = 0$
	$\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = 0$	

Magnetostatic equations imply **B.** *grad p* = 0, **J.** *grad p* = 0

If *grad p* is not zero, magnetic field lines must lie on nested toroidal surfaces of constant *p* (or Klein's bottles)

A. Cerfon lecture

Quasi-symmetry: |B| is symmetric even if B is not

- Can you actually make a non-symmetric **B** with symmetric |**B**|?

- Can you start with a vacuum field with **∇**×**B** = 0 to eliminate most of the internal current in a plasma with finite pressure?

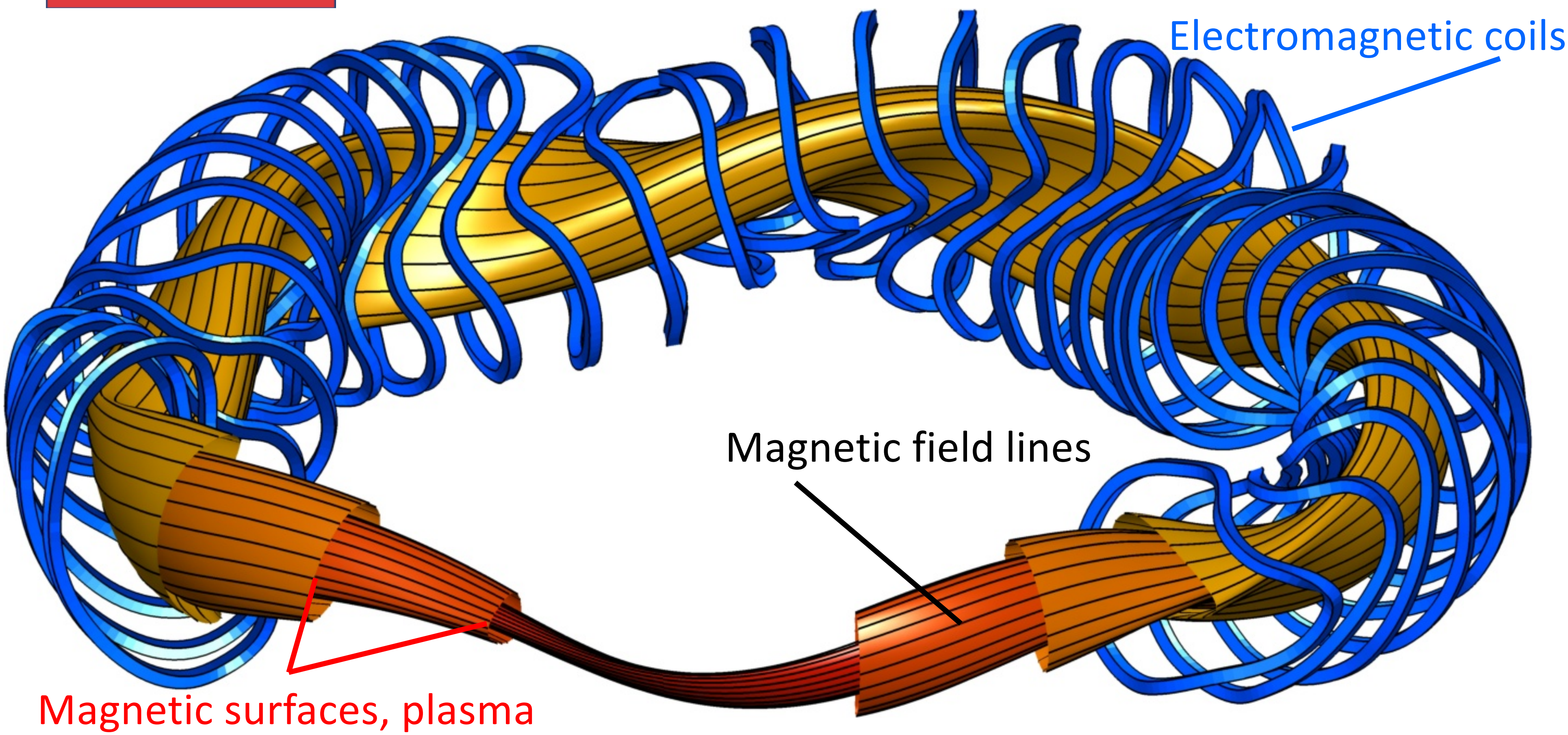
M. Landreman lecture

Example of very nonaxisymmetric magnetic confinement: Wendelstein 7-X (Germany)

Science

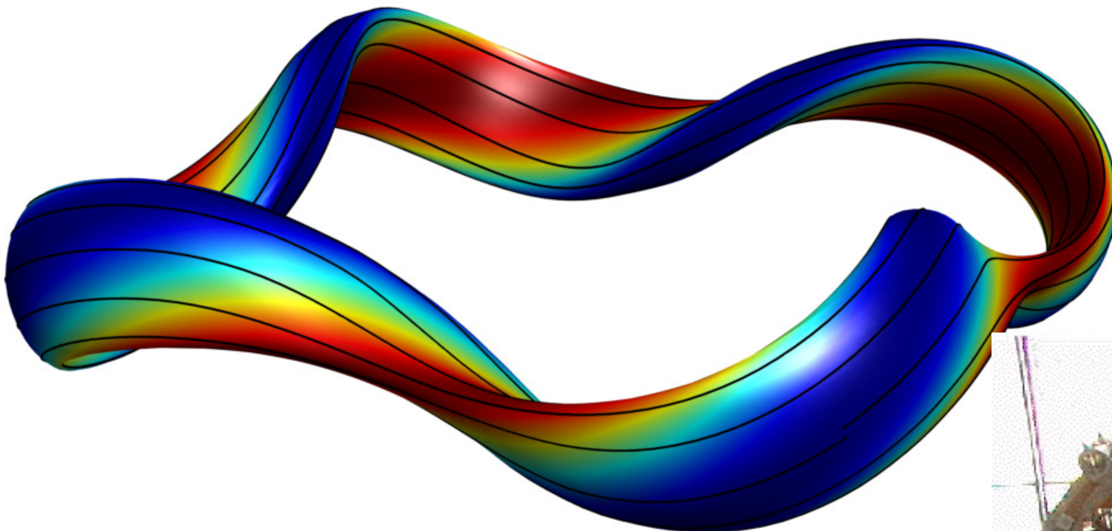
Oct 21, 2015

30 minute plasmas eventually

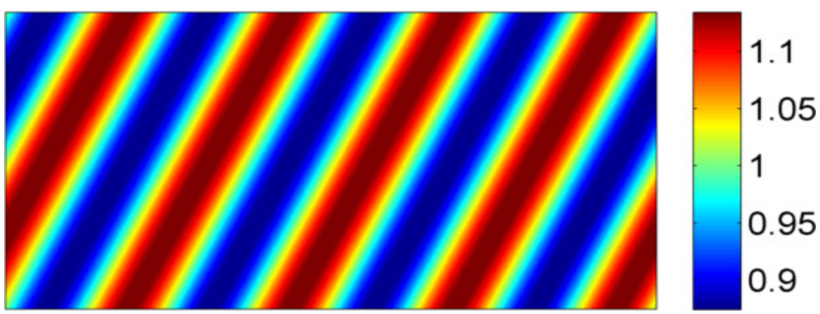


T. Pedersen and E. Stenson lecture

HSX:
Helically Symmetric eXperiment
(University of Wisconsin)

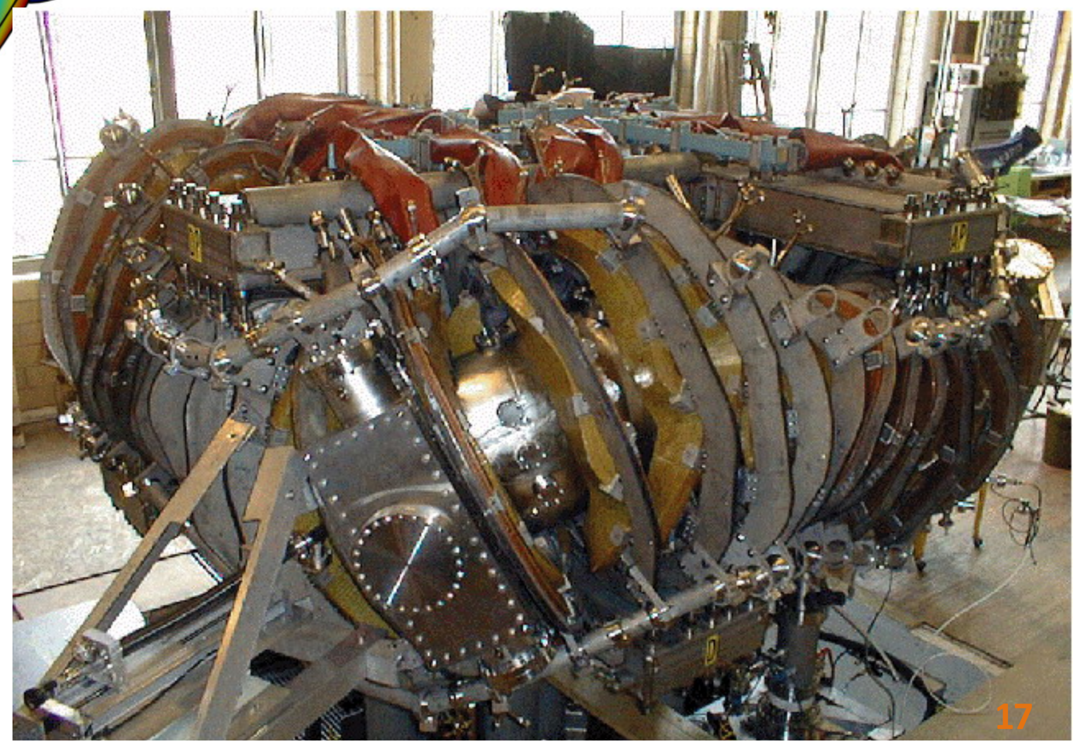


Magnetic field magnitude |B| in Tesla



θ (short way around)

φ (long way around)



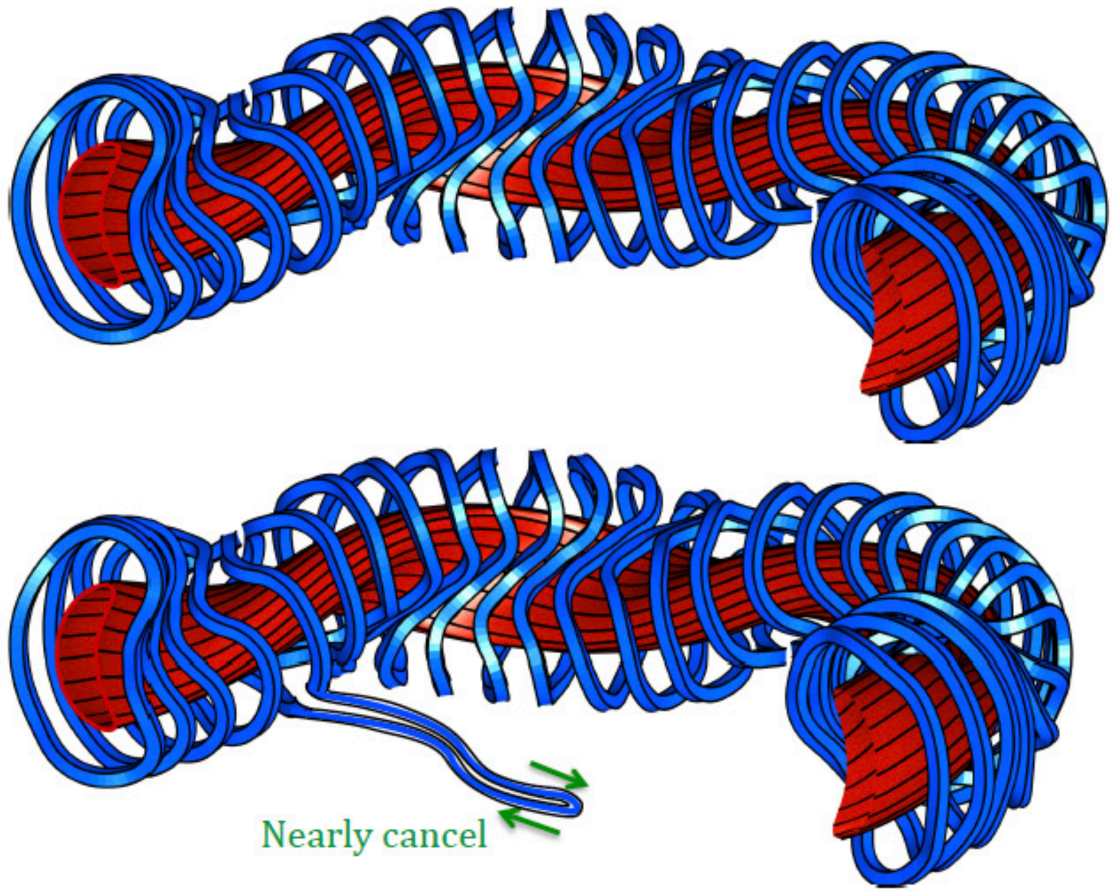
From magnetic surfaces to coils

- Optimization with respect to particle orbits, equilibrium and stability produces a desired 3D magnetic field (with hidden symmetries) that defines the stellarator magnetic surfaces.
- Coil design: Finding coil shapes that produce a given **B** in the confinement volume is an ill-posed problem.
  - There is no unique solution – the same plasma shape can be created from different coil sets. This can be exploited fruitfully.
  - However, it is also a problem that has its challenges: The further away one places the current (the coils) the larger the current and the curvier the coils.
  - Optimum stellarator design becomes a trade-off between the different constraints:
    - Physics optimization, coil curvature, coil tolerances, distance between coils and plasma, stresses in the coils.

S. Hudson lecture

Calculating currents that produce a given B is an ill-posed problem

2 very different coil shapes can produce nearly the same **B** in the confinement region.

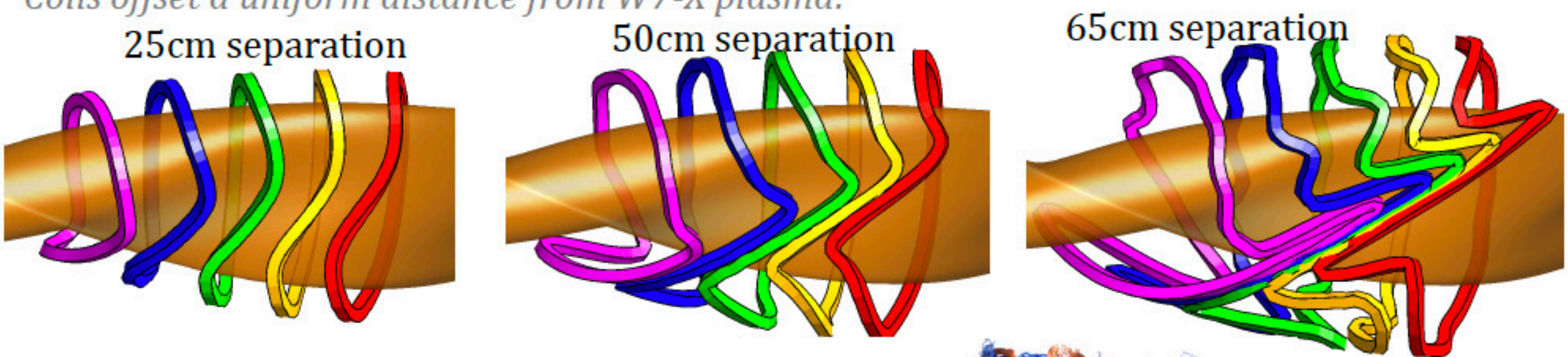


Courtesy: M. Landreman

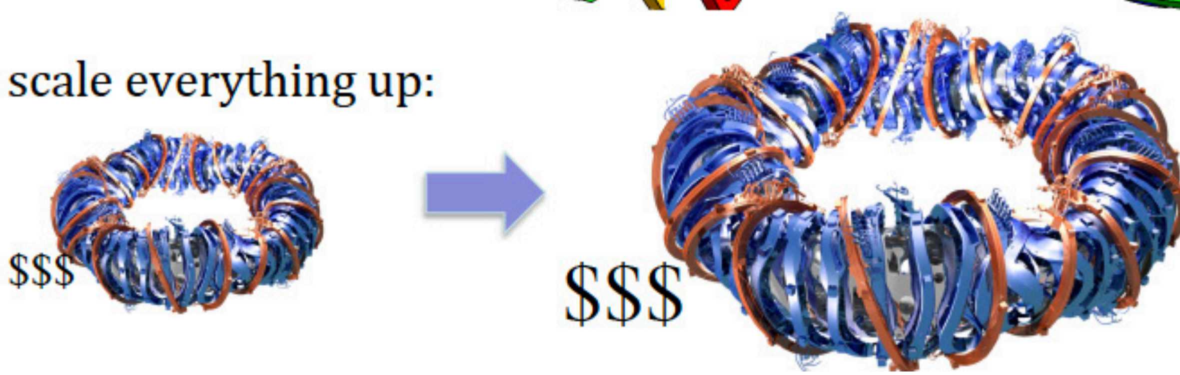
In a reactor, a blanket is needed between coils and plasma to absorb neutrons

But at fixed plasma shape & size, coils shapes become impractical if they are too far away:

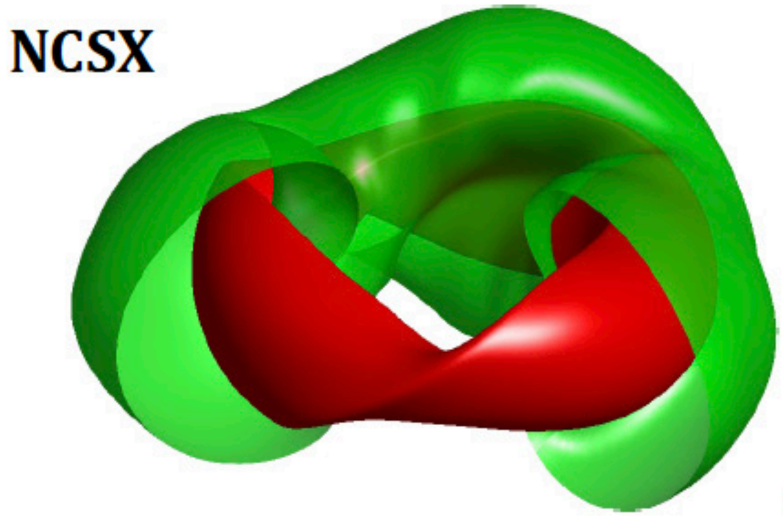
Coils offset a uniform distance from W7-X plasma:

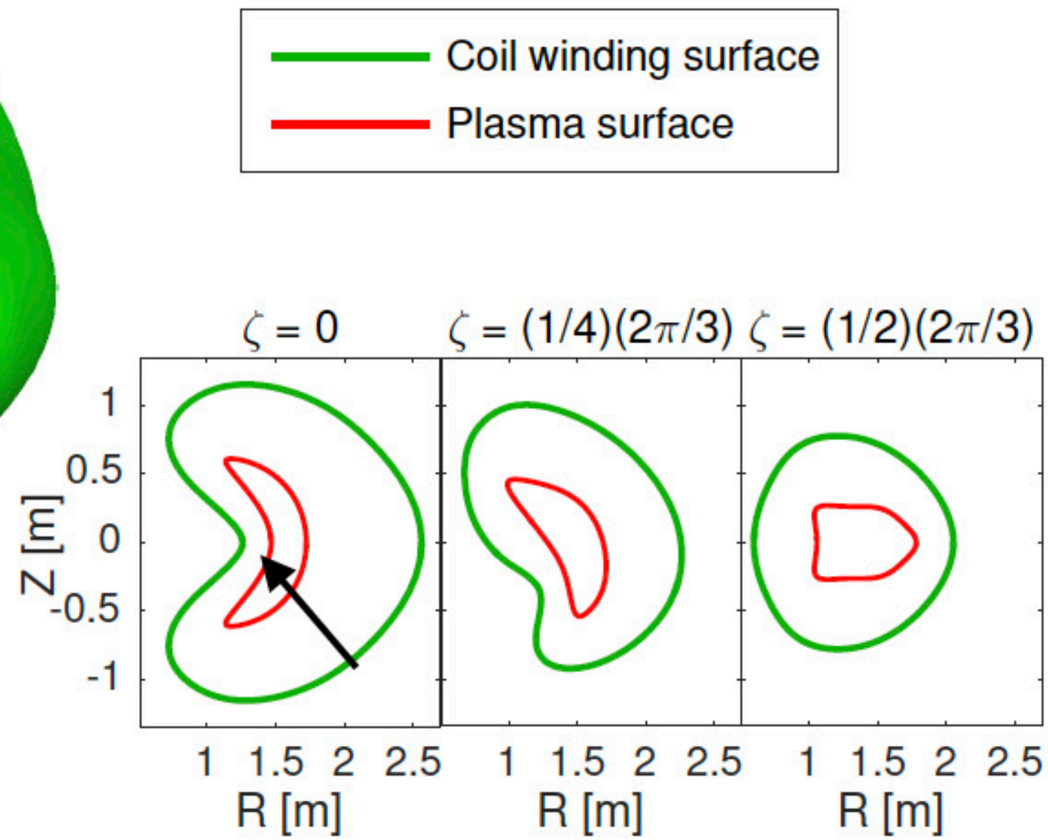


So we must scale everything up:



Why is it so hard to get coils far from *concave* flux surfaces?





Understanding Laplace and Biot-Savart has big payoff for fusion

Between plasma and the coils, the equations are just

$$\mathbf{B} = \nabla\Phi, \quad \Delta\Phi = 0.$$

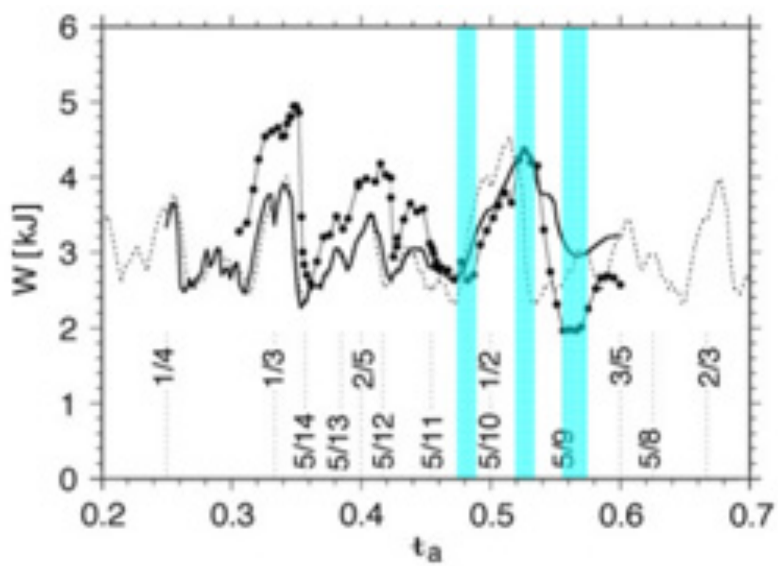
Or, Biot-Savart law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{r}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}$$

$\mu_0 = \text{a constant} = 1.26\times10^{-6} N/A^2$, $\quad I = \text{coil current}$

How do we find robust stellarator configurations?

- Properties of the magnetic field are preserved under realistic deviations constrained by engineering capabilities. Include realistic uncertainties within the optimization calculation. Avoid getting stuck in a non-robust optimum
- *Problem*: Stellarator performance is not a very smooth function of control parameters



M. Hirsch et al., PPCF **50** (2008) 053001

Stochastic optimization is a way to increase tolerances

single optimization	$\longrightarrow$	stochastic optimization

$$\min_{x \in X} f(x) \quad \longrightarrow \quad \min_{x \in X} \left\{ F_N(x) := (N+1)^{-1} \sum_{i=0}^{N} f(\xi^i(x)) + \omega \cdot \sigma \right\}$$

N Sample Size

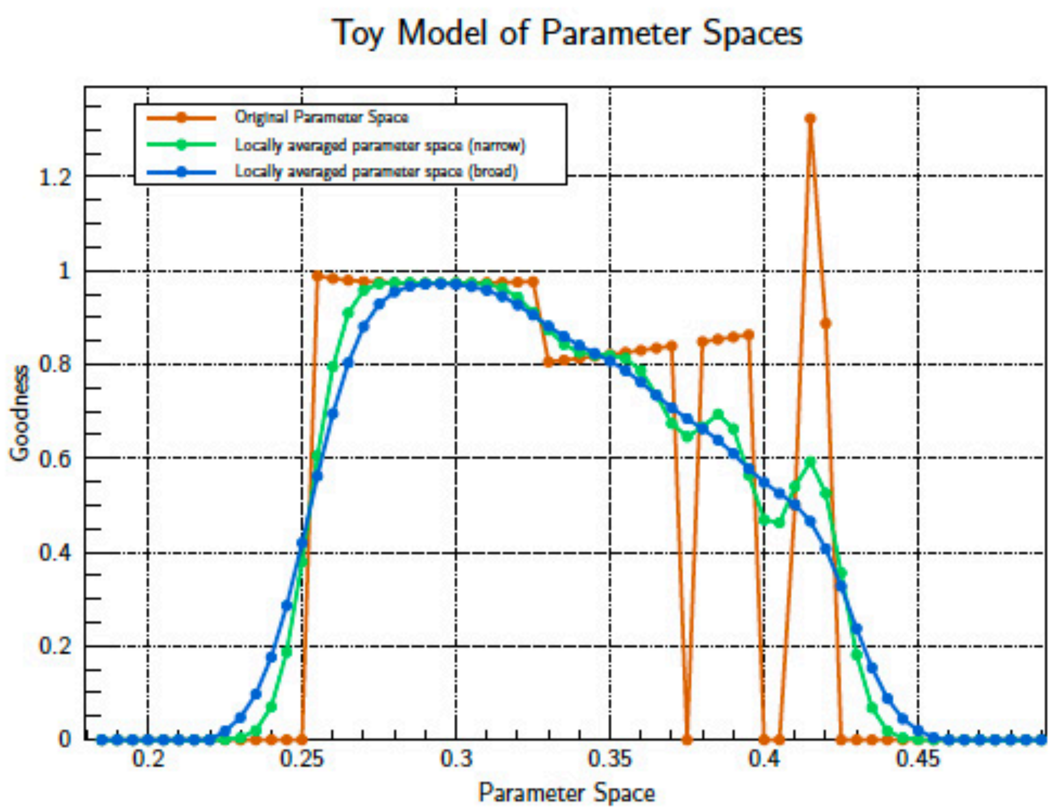
$\xi^0(x) = x$ Unperturbed coil set - leading configuration

$\{\xi^i(x)\}_{i=1,\ldots,N}$ Samples - neighboring coil sets

G. Stadler lecture

Stochastic optimization

Gaussian Distribution



Courtesy: J.-F. Lobsien and T. S. Pedersen

A stellar(ator) comeback?

- These computations are now feasible because of advances in supercomputing power, and algorithmic development. (**S. Henneberg lecture**)
- Deep Learning methods can be especially useful.
- 3D engineering design and manufacturing is becoming standard in industry.
- High-precision metrology equipment is now commonplace.
- We can measure the as-built magnetic topology to the required accuracy.
- Can we exploit new materials to realize high magnetic fields? Or bend the cost curve by using cheaper materials?
- In all of this, our understanding has matured considerably, but foundational mathematical questions remain to be answered.

Some more mathematical questions

- Magnetic field line flow produce a sea of good surfaces and chaos. *How do we quantify non-integrability of field lines and control it? How well do particles track field lines? How are particles transported across "cantori"*?

- Magnetohydrodynamics (MHD) describes the magnetized plasma as a fluid. The 3D MHD problem is not known to be globally well-posed even with dissipation added (similar to the 3D Navier-Stokes problem in hydrodynamics---a Millenium Prize (or Clay) problem). *What are the special solutions enabled by hidden symmetries? The premise of optimum design is that a set of coils with special geometric features can create an environment in which a coupled solution exists without blow-up.*

- *Formulate the design problem as a constrained, risk-averse, multi-objective stochastic optimization problem.* Seek Pareto-optimal solutions. *This cannot be done in a physics-agnostic way.* Existing stochastic optimization methods are not generally very good for addressing high-dimensional problems.

Thank you for your attention.

Questions?